PHY 221 Lab 04-7: Uniform Circular Motion

Objective

In this experiment, you will measure and graph the x-position, y-position, and angle as a function of time for a toy airplane flying in a circle at constant speed. You will use the data to determine the airplane's linear speed, angular speed, and path radius. In addition, you will learn how to use video analysis software to measure position as a function of time for any object.

Introduction to Video Analysis

Video analysis software makes it easy to measure position coordinates (both x and y) and time for an object. After defining the scale and the coordinate system, you click on the object. The software shows a dot where you clicked and advanced the video. The software also measures time because it knows that the video is recorded at 30 frames per second. Thus, whenever you advance the video, time advances 1/30 s.

Suppose the an object moves counterclockwise along a circular path. The picture below shows an object at intervals of 1/30 s between the first image A and the last image F.

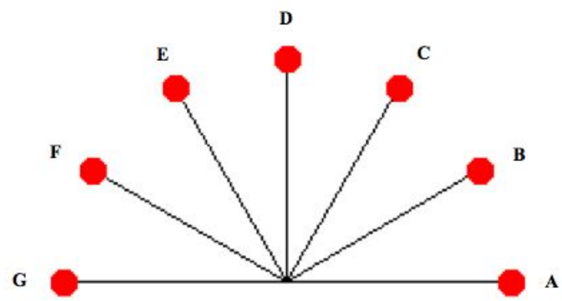

Figure 1:

Suppose that we define $t = 0$ to occur at the first position of the ball. On the picture, label the time t for each subsequent position of the ball.

If you were to graph the x-position of the object as a function of time, what do you think the graph would look like? (No numbers are needed, just a qualitative sketch.)

The x-position of the object for each image is shown below.

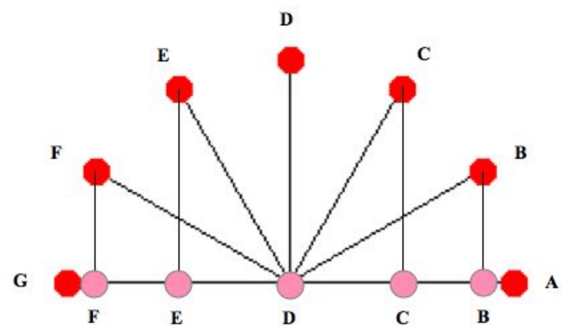

Figure 2:

If you just look at the x-position of the object, what does its motion remind you of?

The radius of the circle is R. We can calculate the object's x and y position at any instant. Let's look at the object when it is at position C. The triangle showing the object, its x-position, and its y-position is shown below.

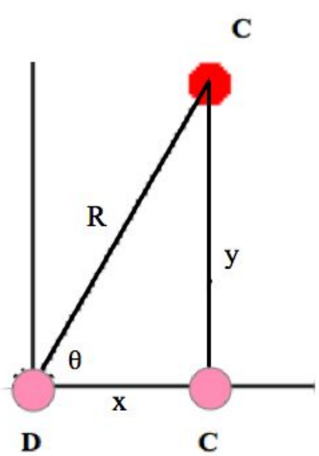

Figure 3:

Using this angle, the x-position and y-position can be calculated as

$$x = R\cos(\theta) \tag{1}$$

$$y = R\sin(\theta) \tag{2}$$

For an object moving in circular motion with a constant speed, the angle θ that the object makes with the +x axis changes at a constant rate. The rate that the angle changes is called the *angular speed*. To calculate angular speed, you measure how much it turns ($\Delta\theta$) and divide by the time interval. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} \tag{3}$$

If θ is the angle at any instant t and if θ_0 is the initial angle at $t = 0$, then

$$\omega = \frac{\theta - \theta_0}{t} \tag{4}$$

$$\theta = \omega t + \theta_0 \tag{5}$$

Thus, the x-position and y-position of the object at the clock reading t is

$$x = R\cos(\omega t + \theta_0) \tag{6}$$

$$y = R\sin(\omega t + \theta_0) \tag{7}$$

As a result, the x-motion and y-motion each resemble simple harmonic motion.

The *linear speed* of the object is distance traveled per second. It's easiest to consider one complete rotation. The distance traveled around a circle is the *circumference*. The time for one revolution is the *period*. Thus, the linear speed of the object is

$$|\vec{v}| = \frac{2\pi R}{T} \tag{8}$$

The angular speed during one revolution is $\omega = 2\pi/T$. Therefore, we can write the linear speed as

$$v = \omega R \tag{9}$$

as long as ω is in units of rad/s. It is typical to drop the magnitude symbol and vector symbol and write the speed $|\vec{v}|$ as v.

In Figure 1, the time interval between positions is 1/30 s, and the angle it turns, between successive positions, is 30°. What is the angular speed in degrees per second?

Often, we use units of radians instead of degrees when measuring angles. 30° is $\pi/6$ radians. What is the angular speed in radians per second?

Experiment

1. Open a web browser and go to our class web site. Click the link to `videos`. Right-click on the link to `04-7-airplane.mov` and save it to your desktop.

2. Open the Logger Pro software.

3. Use Insert→Movie... to import your video.

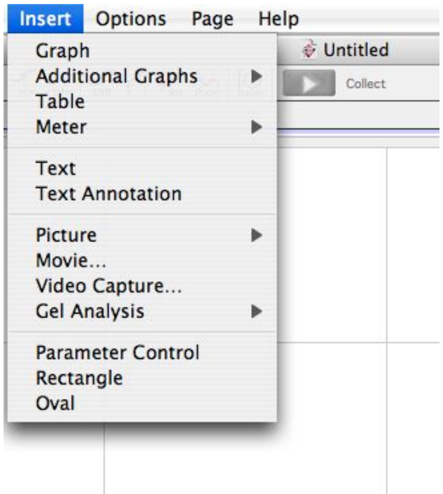

Figure 4: Insert Movie

4. If the video is too small to see, click once on the video, grab the corner of the video window, and expand.

5. At this point, it's nice to lay out the video, data table, and graph so that you can clearly see everything. Go to Page→Auto Arrange... to organize the screen.

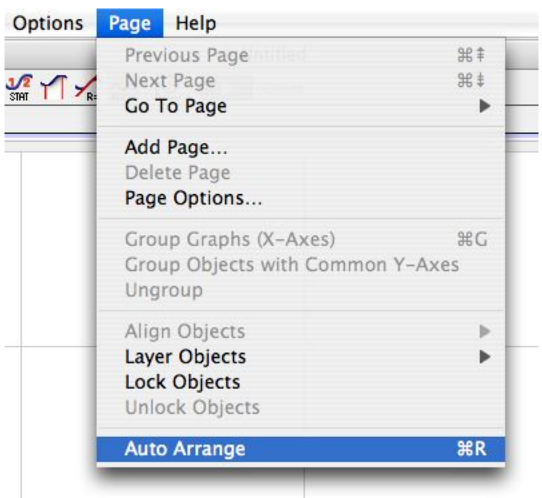

Figure 5:

6. Note the video controls at the bottom of the video pane. Go ahead and play the video, step it forward, backward, etc. in order to learn how the video controls work.

Figure 6:

7. Navigate to the first frame of the video.
8. You now need to define the origin of the coordinate system. Click the icon in the bottom right corner of the video pane (the icon with three red dots) in order to expand the sidebar used for video analysis.

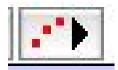

Figure 7:

9. In the sidebar of the video pane, click the appropriate icon to show the coordinate system (you can hover the mouse over each icon to see what they do).

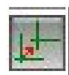

Figure 8:

10. Click and drag on the video to place the origin of the coordinate system at the location where you would like to define (0,0). For this video, place the origin at the black dot near the center, toward the top of the video. This dot is the head of a nail that is holding the pivot to the ceiling.

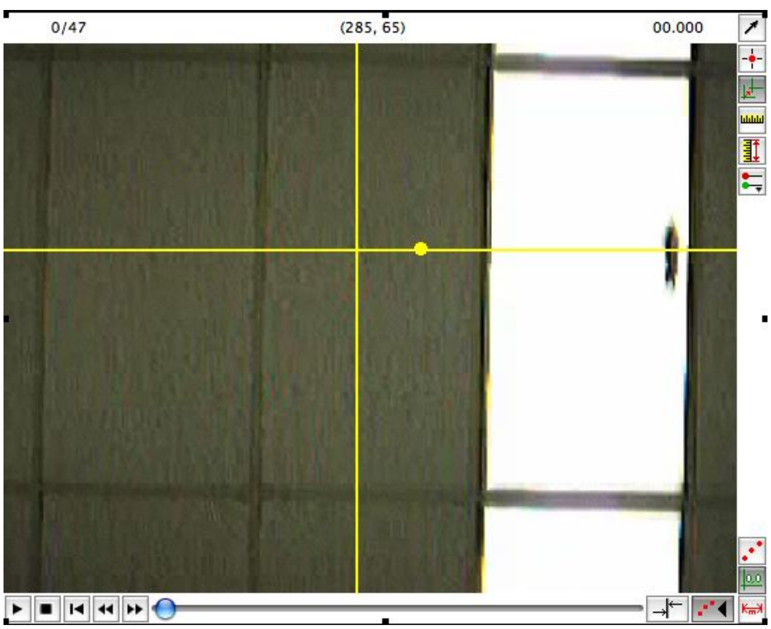

Figure 9:

11. Now, you must calibrate distances measured in the video. In the sidebar, click on the icon of the ruler to set the scale for the video.
12. Click and drag to draw a green line across the width of two ceiling tiles. Enter the actual distance in the resulting pop-up box.The width of one ceiling tile is 0.6 m, so the width of two ceiling tiles is 1.2 m.

Figure 10:

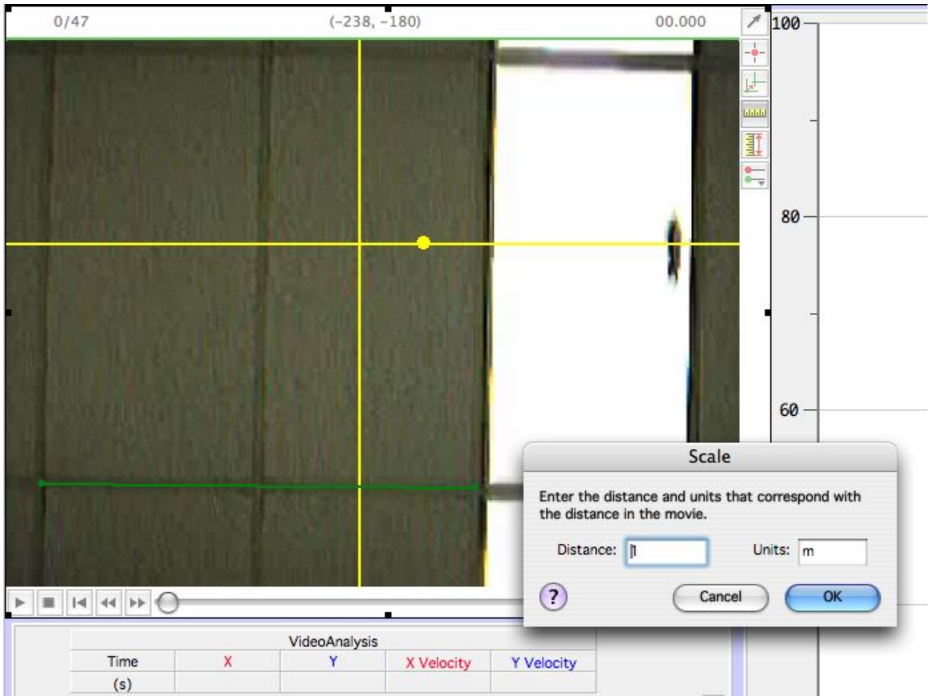

Figure 11:

13. You are ready to add markers to the video to mark the position of the airplane. First, let's not show the coordinate system and scale. It's too distracting. So, click each of the two icons in the lower right corner of the video pane. These icons are used to hide or show the coordinate system and scale.

Figure 12:

14. To add markers, first click on the icon with the red dot and cross hairs. You will now be in an editing mode to add points.

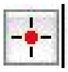

Figure 13:

15. It's important to choose a point on the airplane that you can see in most of the frames of video (except when it disappears off the screen). In this case, it's best to choose the black window of the airplane, near the thickest part of the body of the airplane. This part is fairly easy to identify. The important thing is to be consistent. DO NOT CLICK ON DIFFERENT PARTS OF THE AIRPLANE. Always choose the same part, whether at its center, or its head, or its tail.

Now, click on the point that you choose. You should notice that a marker appears at the position of the airplane where you clicked and that the video advances one frame.

16. Again click on the same part of the airplane to mark its next position. Again, the video will advance.
17. Continue marking the airplane. If the airplane disappears from the video, use the video controls at the bottom of the video to advance the video until the airplane appears again. Then, continue marking the airplane. Your video should like the picture shown below.

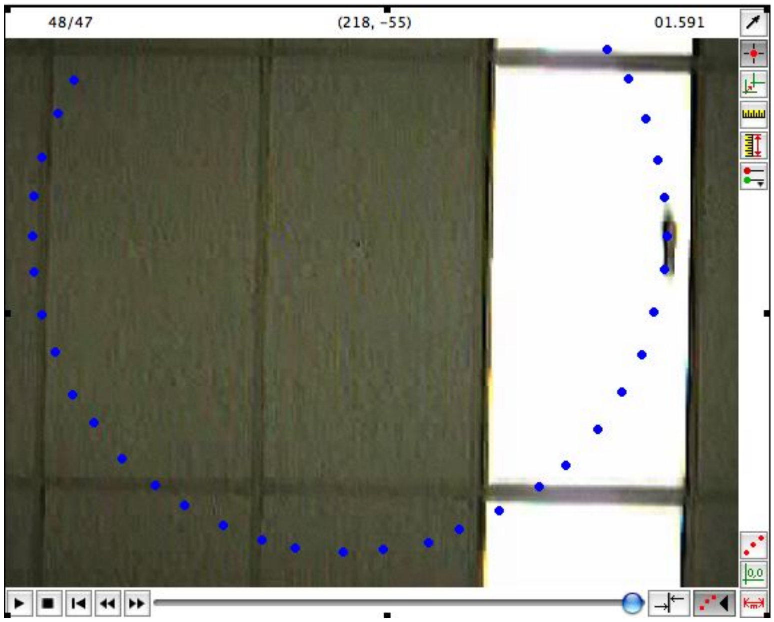

Figure 14:

You are finished collecting data. Now, we will analyze the data.

Analysis

- Click the graph to bring it to the front.
- By default, it will show both the x and y coordinates of the airplane.
- Let's change what is being plotted. First, click the label on the vertical axis. By default, both X and Y are shown. When you click this label, you will have a menu that you can select what is being plotted. Select Y for right now.
- In a similar way, click the label of the horizontal axes and select X.

What do you notice about this graph of Y vs. X?

- Now change the vertical axis to X and the horizontal axis to time.

- This is called a cosine curve. Go to Analyze→Curve Fit. Click the button Define Function... . Enter the function `A*cos(B*t+C)+D` in the box as shown. Click OK. You will see a preview of the curve fit. Again click OK to return to the graph.

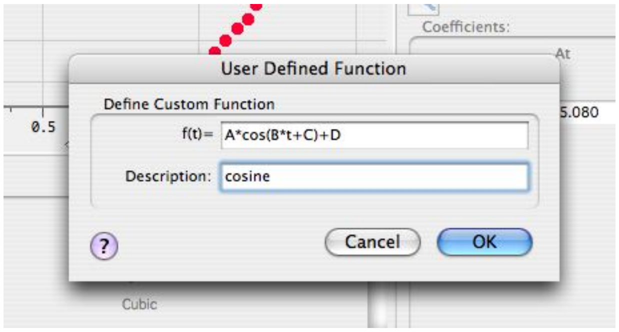

Figure 15:

Record the best-fit function for your curve fit.

- Change the vertical axis label to Y to view the Y vs. time graph. Using the same procedure as before, fit the function $A * sin(B * t + C) + D$ to the graph. This curve is a sine function.

Again record the values of the constants: A, B, C, and D for your curve fit.

By examining the curve fits, we can now determine the radius, period, angular speed, and linear speed of the airplane.

What is the radius of the circle?

What is the airplane's angular speed, in rad/s and deg/s? (The airplane flies $\Delta\theta = 2\pi$ rad in one revolution.)

What is the period of the airplane's motion? (Period is the time interval for one cycle.)

What is the airplane's linear speed, in m/s? ($v = R\omega$)

Uniform Circular Motion – θ as a function of time

The angle θ is calculated by

$$\tan(\theta) = \frac{y}{x} \tag{10}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{11}$$

1. To calculate θ for each marker, go to Data→New Calculated Column... For Name, type `theta` and for Short Name, enter `theta`. For the unit, type `rad`. For the function, type `atan2("Y","X")` which is the computer function for arc-tangent (or inverse-tangent) of y divided by x. The arguments are y and x respectively so that the program can return an angle from 0 to 2π radians.
2. Change the vertical axis label to theta to plot a graph of θ as a function of time.
3. You will notice that the resulting curve is linear.
4. The shape of the curve is a bit unusual because the inverse tangent function computes angles from $-\pi/2$ to $\pi/2$.
5. Select those data points that are 0 to π.
6. Go to Analyze→Linear to fit a line to the data.

What is the slope of the line and what does it tell you?

Errata

In this exercise, we used the ceiling tiles to calibrate the scale of the video. However, the airplane hangs much lower than the ceiling. As a result, it is closer to the video camera. Therefore, measurements of radius and linear speed will not be accurate. We need a meterstick that hangs from the ceiling at the same distance as the plane of the motion of the airplane (i.e. "the plane of the plane.")