PHY 221 LAB 05-2: Energy Principle for a Particle

Energy is like a bank account. The energy of a system only changes when you put energy into or take energy out of the system. How can energy be put into or taken out of a system? Via forces! Forces put energy into or take energy out of a system by doing work on the system.

Work

Suppose that the system is merely a single particle. The work done by a force on a particle depends on the force and the displacement through which it acts. Note that only force components parallel to the displacement through which the force acts can do work.

Work = (Component of force parallel to displacement) * (Magnitude of displacement through which it acts) \( W = f_1 \cdot \Delta \vec{r} \) (1)

In general, if the force is not constant or if the displacement is not along a straight line, then you have to calculate the work done by the force on the particle during a small displacement, and then sum the work along the path of the particle. Mathematically, this is expressed:

\[ W = \int_{i}^{f} \vec{F} \cdot \Delta \vec{r} \] (2)

where the integral is done from point \( i \) to point \( f \) along the path of the particle’s motion. If the force is constant and the path is a straight line, then

\[ Work = F \cos \theta |\Delta r| = F_x \Delta x + F_y \Delta y + F_z \Delta z \] (3)

where \( \theta \) is the angle between force \( \vec{F} \) and displacement \( \Delta \vec{r} \) (draw the vectors tail to tail so you can figure out what the angle is between them).

On a \( F_x \) vs. \( x \) graph, the work done by that component of the force is \( \int_{x_i}^{x_f} F_x \, dx \), which is the area under the curve during the given interval.

Conservation of energy

The total work done on the particle is equal to the energy put into or taken out of the particle. (Just think of the bank account analogy.) If we measure the energy of the particle before work is done and the energy of the particle after work is done, then the energy put into or taken out of the system is the change in the energy of the system.

\[ W_{total} = \Delta E \] (4)

Because the rest energy of the particle doesn’t change, then the change in the energy of the particle is simply its change in kinetic energy.

\[ W_{total} = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \] (5)

This principle is called the work-energy theorem is simply Conservation of Energy applied to a single particle.

When many forces do work, then the total work done on the particle is the sum of the work done by each force.
If the work done on a particle is positive, does the particle speed up, slow down, or have a constant speed? Explain your reasoning.

If the work done on a particle is negative, does the particle speed up, slow down, or have a constant speed? Explain your reasoning.

**Experiment**

1. Set up a level track and obtain a hoop spring, bracket, force sensor, and cart.
2. Measure the mass of the cart.
   
   Record the mass of the cart.

3. Attach the force sensor to the bracket, and attach the hoop spring to the force sensor.
4. Set up the motion detector on the opposite end of the track from the spring. You will use it to measure the position and speed of the cart.
5. Check that the motion detector is working by collecting data and moving the cart back and forth.
6. With the cart just barely touching the hoop spring, zero the force sensor using the Zero button.
7. With your hand on the cart, compress the spring a few cm. (Do not over compress the spring so that you do not damage it.)
8. While holding the cart at rest, with the spring compressed, zero the motion detector.
9. Click Collect to begin collecting data and release the cart from rest.
10. Click the Stop button after it is launched from the spring and catch the cart before it hits the motion detector.

We will analyze the motion of the cart while it is in contact with the spring and is accelerating.
11. First, graph force vs. position. Observe the part of the graph where the spring is in contact with the cart and the cart’s position is changing. Highlight this part of the graph. Go to Analyze → Integral (or use the Integral icon in the toolbar) to measure the area under the force vs. position graph. Record the area under the curve.

   What is the work done on the cart by the spring?

12. Second, graph the cart’s velocity vs. time. You should see it increase from zero to an approximately constant value. (You may notice that it eventually decreases due to friction.) Measure the final speed of the cart (after it has left the spring), by highlighting the portion of the velocity vs. time graph that is constant after it has left the spring, clicking the STAT button, and measuring the average value of the velocity of the cart.

   What is the “final” speed of the cart? What is the “final” kinetic energy of the cart?

   Apply Conservation of Energy to the cart. Is the work done on the cart equal to the change in the total energy of the cart?

   Calculate the \% difference between the work done on the cart and the change in the total energy of the cart by calculating the difference and dividing by the average. Express your answer as a percentage.

13. Repeat the above measurements and calculations for a total of 5 trials.
Application

1. Suppose that a 0.5-kg cart is traveling toward a spring with a speed of 1.5 m/s when it collides with it. If the stiffness of the spring is 10 N/m, what is the maximum amount the spring will compress during the collision?

2. For the previous question, what will be the speed of the cart after it rebounds from the spring (after the collision)?

3. Suppose that you repeat the experiment with the cart and spring. However, you incline the track at an angle of 30° with respect to the horizontal. You use a spring of stiffness 20 N/m. You compress it 4 cm, and you release the cart from rest. If the mass of the cart is 0.5 kg, what is the speed of the cart when it leaves the spring?

4. For the cart in the previous question, what is the maximum distance it will travel along the track?