

PHY221 Lab-05-5: Energy Diagram

Objective: Write a VPython program that simulates a planet with the mass of Earth orbiting a star with mass of Sun. Calculate and graph its potential energy, its kinetic energy, and its total energy as a function of distance between the Sun and planet.

Review

The gravitational potential energy of a two-body system is:

$$U = -G \frac{m_1 m_2}{r} \quad (1)$$

where $r = |\vec{r}|$ and $\vec{r} = \vec{r}_2 - \vec{r}_1$, the relative position vector of one body with respect to the other body. In this case, the central body (the star) is about a million times more massive than the planet. As a result, it's wobble, due to the gravitational force by the planet on the star, is negligible and we can neglect the change in kinetic energy of the star. The kinetic energy of the system is then

$$K = 1/2 m v^2 \quad (2)$$

where m is the mass of the planet and v is the speed of the planet. The total mechanical energy of the system (which does not include rest energy) is

$$E = K + U \quad (3)$$

Circular Orbit

For a circular orbit, such as Earth in Fig. ??, the distance r is constant and the speed v is constant.

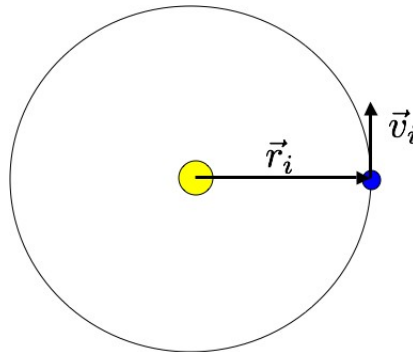


Figure 1: Earth's initial position and velocity in the simulation

The planet's speed is

$$v = \sqrt{\frac{GM}{R}} \quad (4)$$

where M is the mass of the star (Sun) and R is the radius of the circular orbit. The total energy becomes

$$\begin{aligned} E &= K + U \\ &= 1/2 m v^2 - \frac{GmM}{r} \\ &= -\frac{1}{2} \frac{GmM}{r} \end{aligned}$$

Therefore, the potential energy is constant and the kinetic energy of the system is constant for a circular orbit. A graph of U vs r and K vs r is not very interesting since there will only be one datapoint on the graph.

Elliptical Orbit

However, for an elliptical orbit or an open orbit (such as a parabola or hyperbola), the distance between the planet and star changes and the graph of potential and kinetic energy as a function of distance are much more interesting. These orbits are what you will study in this activity.

Writing your program

1. Start with the simulation of Earth's orbit that you wrote in a previous lab. If you can't find it or if you have deleted it, then you will have to ask a classmate to send you their code so that you can start with theirs.
2. The first three lines import the necessary libraries. You will need the graph library to plot your energy graph. Thus, make sure that the first three lines of the program are:

```
from __future__ import division
from visual import *
from visual.graph import *
```

3. The star (Sun) should be at the origin and the planet (Earth) should be at a distance of 1 AU (1.5×10^{11} m) from the star on the +x axis.
4. Check the constants for the mass of Sun and the mass of Earth to make sure that they are correct.
5. Define a constant for R , the radius of Earth's orbit if it is in circular motion.

```
R = 1.5e11
```

6. Define the initial speed of the planet (Earth). For an elliptical orbit, let's use 1.1 times the speed for a circular orbit ($1.1v_{circle}$).

```
vi=1.1*sqrt(G*sun.m/R)
```

7. Define the initial velocity of the planet to be $\langle 0, v_i, 0 \rangle$.
8. Check that the initial momentum is calculated correctly as the mass of the planet times its initial velocity.
9. Before the while statement, create a graph that will have three plots for U, K, and E.

```
UGraph=gdisplay(title="U vs. r", xtitle='r (m)',ytitle='U (J)', x=450, y=0, width=400, height=200)
UPlot=gcurve(color=color.cyan)
EPlot=gcurve(color=color.yellow)
KPlot=gcurve(color=color.white)
```

10. Inside your while loop, after r is calculated and before the gravitational force is calculated, calculate the energies. The reason you need to calculate energy before the force, velocity, and momentum is that you don't want to calculate kinetic energy after the speed has been updated and potential energy before the speed is updated. You have to be consistent and calculate the kinetic and potential energy at the same points in the orbit.

Be sure to use the correct variable names for your star and planet. Here are examples of the calculations.

```

U = -G*sun.m*earth.m/r
K = 1/2*earth.m*mag(earth.v)**2
E = K+U

```

11. At the end of the while loop, you must add data points to the graph.

```

UPlot.plot(pos=(r,U))
EPlot.plot(pos=(r,E))
KPlot.plot(pos=(r,K))

```

12. Run your simulation and view the resulting energy diagram.

Application

- For a circular orbit with $v = \sqrt{\frac{GM}{R}}$, what does the energy diagram look like? Sketch the energy diagram and the orbit.
 - Is the total energy positive, negative or zero?
- For an elliptical orbit with an initial speed of 0.8 times the speed for a circular orbit, what does the energy diagram look like? Sketch the energy diagram and the orbit.
 - Which point in the energy diagram corresponds to the location of the planet when it is at perihelion (closest point to Sun)?
 - Which point in the energy diagram corresponds to the location of the planet when it is at aphelion (furthest point from Sun)?
 - Is the total energy positive, negative or zero?
- For an elliptical orbit with an initial speed of 1.2 times the speed for a circular orbit, what does the energy diagram look like? Sketch the energy diagram and the orbit.
 - Which point in the energy diagram corresponds to the location of the planet when it is at perihelion (closest point to Sun)?
 - Which point in the energy diagram corresponds to the location of the planet when it is at aphelion (furthest point from Sun)?
 - Is the total energy positive, negative or zero?
- Sketch the initial speed to be exactly the escape speed, $\sqrt{2}$ times the speed for a circular orbit. Sketch the energy diagram and the orbit.
 - Which point in the energy diagram corresponds to the location of the planet when it is at perihelion (closest point to Sun)?
 - Is the total energy positive, negative or zero?
- Sketch the initial speed to be greater than the escape speed, perhaps 1.5 times the speed for a circular orbit. Sketch the energy diagram and the orbit.
 - Which point in the energy diagram corresponds to the location of the planet when it is at perihelion (closest point to Sun)?
 - Is the total energy positive, negative or zero?
- If the total energy of the system is positive, is it bound or unbound?
- If the total energy of the system is negative, is it bound or unbound?
- Suppose that a system consists of satellite orbiting Earth with a total energy of 7800 J. How much energy is needed to make it escape its orbit around Earth? (i.e. become unbound)