Question (1220004)

Change in momentum of Earth in 1 day during its orbit around Sun

Earth orbits Sun in a nearly circular orbit with a speed $3.0 \times 10^4$ m/s. Suppose that at a certain instant, Earth’s velocity is in the $-x$ direction. At this instant, the force by Sun on Earth is $< 0, -3.6 \times 10^{22}, 0 >$ N. What will be Earth’s momentum 1 day later? During this time interval, the only significant force acting on Earth is the gravitational force by Sun on Earth.

![Diagram of gravitational force by Sun on Earth.](image)

**Figure 1:** The gravitational force by Sun on Earth.

Solution

According to the **Momentum Principle**, the net impulse on Earth causes a change in momentum of Earth according to.

$$\Delta \vec{p} = \vec{F}_{\text{net}} \Delta t$$

The change in momentum of Earth is in the same direction as the net force on Earth, which in this case is simply the gravitational force by Sun on Earth. Use this fact, to sketch the change in momentum of Earth and the final momentum of Earth after 1 day. The change in momentum vector is placed at the head of the initial momentum vector, and the final momentum is drawn from the tail of the initial momentum to the head of the change in momentum.

In the above sketch, it was imperative to know that the change in momentum is drawn in the same direction as the net force on Earth. This comes from the Momentum Principle. Now, substitute the given quantities to calculate the final momentum after 1 day. Be sure to convert 1 day to seconds. Also, look up the mass of Earth in your textbook or an online reference.
Figure 2: The change in momentum of Earth as a result of the gravitational force on Earth by Sun.

\[
\vec{p}_f - \vec{p}_i = \vec{F}_{net}\Delta t
\]

\[
\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t
\]

\[
= m_{Earth}\vec{v}_i + \vec{F}_{net}\Delta t
\]

\[
= (6 \times 10^{24} \text{ kg})(< -3 \times 10^4, 0, 0 > \text{ m/s}) + (< 0, -3.6 \times 10^{22}, 0 > \text{ N})(1 \text{ day}) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right)
\]

\[
= (< -1.8 \times 10^{29}, 0, 0 > \text{ kg m/s}) + (< 0, -3.1 \times 10^{27}, 0 > \text{ kg m/s})
\]

\[
= < -1.8 \times 10^{29}, -3.1 \times 10^{27}, 0 > \text{ kg m/s}
\]

Examine the result. Both the x and y components are negative, which is consistent with the picture. Also, note that the y-component is small compared to the x-component. In fact, the angle that this vector makes with the -x axis is

\[
\theta = \tan^{-1} \left( \frac{3.1 \times 10^{27}}{1.8 \times 10^{29}} \right)
\]

\[
= 0.99^\circ
\]

Figure 3: The angle of Earth’s final momentum after 1 day.

This is a very small angle because 1 day is a small time interval compared to Earth’s 365 day orbit. Thus, Earth’s momentum changes in one day, but only by about 1 degree.