Question (1a10001)

Angular momentum of a child running toward a merry-go-round

A 20 kg child runs at a constant speed of 3.0 m/s, in a straight line tangentially to a merry-go-round, and jumps onto the edge of the merry-go-round of radius 2.0 m, as shown below.

Figure 1: A child running toward a merry-go-round.

(a) What is the angular momentum of the child, relative to the center of the merry-go-round, just before landing on the merry-go-round?

(b) What is her angular momentum, relative to the center of the merry-go-round, when she is 10.0 m from the edge of the merry-go-round?

(c) What if she ran in the same direction with the same speed but jumped onto the “lower edge” of the merry-go-round in the picture, causing the merry-go-round to rotate counterclockwise?

Solution

(a) Sketch a picture of the child at the point where she jumps onto the merry-go-round. Show her position and her momentum.

Her angular momentum is

\[
\vec{L}_{\text{initial}} = \vec{r} \times \vec{p} \\
= \vec{r} \times m\vec{v} \\
= \begin{pmatrix} 0, 2, 0 \end{pmatrix} \text{ m} \times (20 \text{ kg}) \begin{pmatrix} 3, 0, 0 \end{pmatrix} \text{ m/s} \\
= \begin{pmatrix} 0, 2, 0 \end{pmatrix} \text{ m} \times (20 \text{ kg}) \begin{pmatrix} 3, 0, 0 \end{pmatrix} \text{ m/s} \\
= \begin{pmatrix} 0, 0, -6 \end{pmatrix} \text{ kg m}^2/\text{s}
\]

It is in the \(-z\) direction, which is consistent with the right-hand rule. If you draw \(\vec{r}\) and \(\vec{p}\) tail to tail, you can see that \(\vec{r}\) rotates clockwise into \(\vec{p}\), which gives the \(-z\) direction. A picture is shown below, but it’s much easier to visualize using your right hand alone.
Figure 2: The child at the instant before she lands on the merry-go-round.

Figure 3: Using the right-hand rule. Clockwise rotation of \( \mathbf{r} \) into corresponds to an angular momentum into the plane.

(b) At a horizontal distance of 10 m from the edge of the merry-go-round, the child’s angular momentum is

\[
\mathbf{L}_{\text{child}} = \mathbf{r} \times \mathbf{p} = \mathbf{r} \times m\mathbf{v} = \langle -10, 2, 0 \rangle \text{ m/s} \times (20 \text{ kg})\langle 3, 0, 0 \rangle \text{ m/s} = \langle -60, 0, 0 \rangle \text{ kg m}^2/\text{s}
\]

Note that it is the same as when she jumps onto the merry-go-round. This is because the perpendicular distance \( r_\perp \) between her momentum vector and the origin is the same, 2 m. As a result, her angular momentum is the same.

(c) Sketch a picture of the situation.

Her angular momentum is
Figure 4: The child at the instant before she lands on the merry-go-round.

\[
\vec{L}_{\text{child}} = \vec{r} \times \vec{p} \\
= \vec{r} \times m\vec{v} \\
= \langle 0, -2, 0 \rangle \text{ m } \times (20 \text{ kg}) \langle 3, 0, 0 \rangle \text{ m/s} \\
= \langle 0, -2, 0 \rangle \text{ m } \times (20 \text{ kg}) \langle 3, 0, 0 \rangle \text{ m/s} \\
= \langle 0, 0, 6 \rangle \text{ kg m}^2/\text{s}
\]

This time, it is in the +z direction which can be verified with the right-hand rule as shown below.

Figure 5: Using the right-hand rule. Counterclockwise rotation of r into p corresponds to an angular momentum out of the plane.