Question (14d0002)

**Perpendicular and tangential forces on a comet in an elliptical orbit.**

A comet orbits a star counterclockwise as shown below.

For each point shown (A, B, C, and D), answer the following questions.

(a) Sketch a vector showing the momentum of the comet.

(b) Sketch a vector showing the net force on the comet.

(c) Sketch the perpendicular component of the net force on the comet.

(d) Sketch the tangential component of the net force on the comet.

(e) At this instant, is the comet speeding up or slowing down?

**Solution**

**Point A**

The momentum of the comet is tangent to the path and in the direction of motion. The net force on the comet is the gravitational force by the star on the comet, since there are presumably no other significant forces acting on the comet. Thus, the net force on the comet is toward the star.

The momentum is in the +y direction and the net force is in the −x direction. Thus, the net force on the comet at this instant is perpendicular to the path. As a result, the tangential component of the net force is zero. Then, according to the momentum principle in the tangential direction

\[
F_{\text{net,\,tan}} = \frac{d|\vec{p}|}{dt} \hat{p}
\]

\[
0 = \frac{d|\vec{p}|}{dt} \hat{p}
\]

\[
\frac{d|\vec{p}|}{dt} = 0
\]
Though this could mean that the magnitude of the comet’s momentum, and therefore speed, is constant, it can also mean that the magnitude of the comet’s momentum is a maximum or minimum since the derivative of a quantity at a maximum or minimum is zero. In this case, at Point A, the magnitude of the comet’s momentum is a minimum; therefore, its speed is a minimum. It is neither speeding up nor slowing down, but rather transitioning from slowing down to speeding up (i.e. the speed is a minimum). You can see this more clearly in a simulation by clicking the “View Simulation” link below.

**View Simulation**

**Point B**

At point B, the net force on the comet is pointed toward the star, and the momentum is in the direction of motion, as shown below.

Make a right triangle with one component of the net force perpendicular to the path and one component tangent to the path. Note that the tangential component of the net force is in the same direction as the momentum. As a result, at point B the comet is speeding up. This will true of any point as the comet travels from point A to point C in its orbit.
Point C

When the comet is at point C, its momentum is in the \(-y\) direction, and the net force on the star (due principally to the gravitational pull of the star) is toward the star, in the \(+x\) direction. The comet’s momentum is quite large at this point, as is the net force, compared to point A for example.

The net force on the comet at point C is perpendicular to the path; therefore, the tangential component of the net force is zero. According to the momentum principle in the tangential direction

\[
F_{net,\text{tan}} = \frac{\dot{p}_\parallel}{\dot{t}} \hat{p}_\parallel \\
0 = \frac{\dot{p}_\parallel}{\dot{t}} \hat{p}_\parallel \\
\frac{\dot{p}_\parallel}{\dot{t}} = 0
\]

In this case, at Point C, the magnitude of the comet’s momentum is a maximum; therefore, its speed is a maximum. The comet is neither speeding up nor slowing down, but rather transitioning from speeding up to slowing down (i.e. the speed is a maximum). You can see this more clearly in the simulation by clicking the “View Simulation” link below.

View Simulation

Point D

At point D, the net force on the comet is pointed toward the star, and the momentum is in the direction of motion \((-x\), as shown below.

Draw a right-triangle with the perpendicular component of the net force perpendicular to the path \((+y)\) and the tangential component parallel to the path. The tangential component is in the \(-x\) direction which is opposite the momentum. Therefore, \(\frac{\dot{p}_\parallel}{\dot{t}}\) is negative, so the comet at this instant must be slowing down.
Figure 5: The comet at point D.