**Question (1530001)**

**Work done on a pole vaulter during a vault**

The path of a pole vaulter is measured using video analysis. The $y$ vs. $x$ graph (which shows the path) of the pole vaulter is shown below.

![Figure 1: The path of a pole vaulter.](image)

The mass of the vaulter is 70.0 kg. He plants the pole when he is at the location $\vec{r}_i = <3.10, 0, 0>$ m, and the pole starts to bend. With the pole mostly bent, the vaulter starts his mostly upward trajectory at the location $\vec{r}_f = <5.32, 0.835, 0>$ m. The average force by the pole on the pole vaulter during this interval is $<-525, 275, 0>$ N. What is the total work done on the pole vaulter during this interval? State any assumptions that you make.

**Solution**

Though the work can be computed numerically by calculating the work done between each data point in the graph, it is perhaps simplest to assume a straight line path between the initial and final position of the interval given in the problem. This displacement is drawn below.

We will make three assumptions: (1) the pole vaulter travels in a straight line; (2) the force by the pole on the pole vaulter is constant; and (3) the pole vaulter can be treated as a particle. The total work done on the system is the sum of the work done by each force on the system.

$$W = W_1 + W_2 + \cdots$$

First, define the system. In this case, the system is the pole vaulter (i.e., the person without the pole). The system, which we are treating as a particle, only has rest energy and kinetic energy, and only its kinetic energy can change during the interval since its mass remains constant.
Next, identify the external forces that do work on the system: (1) the pole and (2) Earth. A free-body diagram shows the forces on the system during the given interval.

The total work done on the system is:

\[ W = W_{\text{grav}} + W_{\text{pole}} \]

Calculate the work done by each force. Starting with the pole, the work done by the pole on the person is
\[ W = \vec{F}_{\text{pole}} \cdot \Delta \vec{r} \]
\[ = F_{\text{pole},x} \Delta x + F_{\text{pole},y} \Delta y + F_{\text{pole},z} \Delta z \]
\[ = (-525 \text{ N})(5.32 - 3.10) \text{ m} + (275 \text{ N})(0.835 - 0) \text{ m} + (0)(0) \]
\[ = -1165 \text{ J} + 230 \text{ J} \]
\[ = -935 \text{ J} \]

Check that the sign makes sense. Sketch the force by the pole and the displacement through which it acts, tail to tail as shown below.

![Figure 4: The force by the pole and the displacement through which it acts.](image)

The angle between them is greater than 90°; therefore, the work done by the pole on the system is negative. \((W = F \cos \theta\) is negative for angles greater than 90°.) As a result, the pole causes the system to lose kinetic energy.

Now, calculate the work done by Earth during the given interval. The gravitational force by Earth on the person is in the \(-y\) direction, thus the work done by the gravitational force is

\[ W = \vec{F}_{\text{grav}} \cdot \Delta \vec{r} \]
\[ = F_{\text{grav},x} \Delta x + F_{\text{grav},y} \Delta y + F_{\text{grav},z} \Delta z \]
\[ = F_{\text{grav},y} \Delta y \]
\[ = (-mg) \Delta y \]
\[ = -(70 \text{ kg})(9.8 \text{ N/kg})(0.835 - 0) \text{ m} \]
\[ = -573 \text{ J} \]

The work done by the gravitational force is negative since the vaulter has an upward (positive) displacement. The total work done on the system is

\[ W = W_{\text{grav}} + W_{\text{pole}} \]
\[ = -573 \text{ J} + -935 \text{ J} \]
\[ = -1508 \text{ J} \]
\[ \approx -1510 \text{ J} \]

According to conservation of energy for a particle
\[ W = \Delta K \]

Thus, since the work done on the system is negative, then the system lost kinetic energy during this interval which means that it slowed down. This make sense because when a running pole vaulter plants the pole and it bends, he necessarily slows down.