Question (1a30002)

Net torque on a rod.

A rod can rotate about an axis perpendicular to the page and through the left end of the rod as shown. The forces shown in the image are applied to the rod. \( \vec{F}_1 \) is applied at the end of the rod, \( \vec{F}_2 \) is applied to the middle of the rod, and \( \vec{F}_3 \) is applied at one-fourth the length of the length of the rod. The length of the rod is 0.5 m. All forces have a magnitude of 75 N.

![Figure 1: Forces applied to a rod.](image)

What is the net torque on the rod?

Solution

The net torque is sum of the torques due to each force on the rod.

\[
\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4
\]

Use the definition of torque to calculate the torque due to each force.

\[
\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1
\]

Sketch and determine the position vector for the location where each force is applied to the rod.

To calculate the torque due to \( \vec{F}_1 \), first write the force in vector notation. Use direction cosines to determine the force components.

\[
F_{1,x} = |\vec{F}_1| \cos(\theta_x) = |\vec{F}_1| \cos(90^\circ + 55^\circ) = (75 \text{ N}) \cos(145^\circ) = -61.4 \text{ N}
\]
Figure 2: Position vectors for each force.

\[
\begin{align*}
F_{1y} &= |\vec{F}_1| \sin(\theta_y) \\
&= |\vec{F}_1| \cos(55^\circ) \\
&= (75 \text{ N}) \cos(55^\circ) \\
&= 43.0 \text{ N}
\end{align*}
\]

Then, take the cross product to calculate torque. Use the symbol \( L \) for the length of the rod.

\[
\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 \\
= (L, 0, 0) \times (F_{1x}, F_{1y}, 0) \\
= (0, 0, L) \\
= (0, 0, (43 \text{ N})(0.5 \text{ m})) \\
= (0, 0, 21.5) \text{ N m}
\]

Calculate the torque due to each of the other forces in the same way. Note that \( \vec{r}_4 = (0, 0, 0) \) and therefore \( \vec{\tau}_4 = (0, 0, 0) \).

\[
\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 \\
= \left(\frac{L}{2}, 0, 0\right) \times (F_{2x}, F_{2y}, 0) \\
= (0, 0, \frac{L}{2}) \\
= (0, 0, (-75 \text{ N})(0.25 \text{ m})) \\
= (0, 0, -37.5) \text{ N m}
\]
\[ \vec{\tau}_a = \vec{r}_a \times \vec{F}_a \]
\[ = \left( \frac{L}{4}, 0, 0 \right) \times (F_{3x}, F_{3y}, 0) \]
\[ = \left( 0, 0, F_{3y} \frac{L}{4} \right) \]
\[ = \left( 0, 0, \left| \vec{F}_a \right| \cos(90 - 70) \frac{L}{4} \right) \]
\[ = \left( 0, 0, (75 \text{ N}) \cos(20)(0.25 \text{ m}) \right) \]
\[ = \left( 0, 0, 15.2 \right) \text{ N m} \]

The net torque on the rod is

\[ \vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 \]
\[ = \left( 0, 0, 21.5 \right) \text{ N m} + \left( 0, 0, -37.5 \right) \text{ N m} + \left( 0, 0, 15.2 \right) \text{ N m} \]
\[ = \left( 0, 0, -0.8 \right) \text{ N m} \]

Compared to each individual torque on the rod, the net torque on the rod is somewhat small. As a result of the net torque, the angular momentum of the rod will change. If the rod starts from rest, the rod will rotate clockwise in the x-y plane due to the net torque that is in the -z direction.