Question (1220003)

Impulse and average net force on a model rocket by the engine

The thrust of a certain model rocket engine as a function of time is shown below. Here, the word “thrust” refers to the force by the exhaust on the rocket as the rocket engine is firing.

![Thrust curve of an Estes Model Rocket Engine](image)

Figure 1: The thrust curve of a certain model rocket engine.

The total area under the curve is 1.84 N s. The total burn time of the rocket engine is 1.24 s. The peak thrust has a magnitude 8.60 N. Since force is a vector, assume that the thrust is in the +y direction.

(a) Calculate the average thrust on the rocket due to the rocket engine.

(b) Compare the average thrust to the peak thrust. Explain why the average thrust is so much smaller than the peak thrust.

(c) If the thrust by the rocket engine on the rocket is the only force acting on the rocket, then what will be the change in the rocket’s momentum during the time interval from $t=0$ to $t=1.24$ s?

Solution

(a) The impulse due to thrust is

$$\text{impulse} = \vec{F}_{\text{thrust}} \Delta t$$

However, this only works if the thrust is constant. In this case, it is not constant. Therefore, one must integrate by finding the impulse $\int \vec{F}_{\text{thrust}} dt$ during each small time interval $dt$ and summing over the entire time interval, the burn time. This is an integral.

$$\text{impulse} = \int_0^{1.24 \text{ s}} \vec{F}_{\text{thrust}} dt$$
This can be found on the graph by measuring the area under the curve for the entire burn time, 1.24 s. This is given to us as 1.84 N s; therefore, the impulse is

\[
\int_{0}^{1.24} F_{\text{thrust}} dt = <0, 1.84, 0> \text{ N s}
\]

To find the average thrust, assume that the thrust is constant over the same time interval. What constant thrust will produce the same impulse as the actual, varying thrust during the same time interval? The result is called the average thrust.

\[
\bar{F}_{\text{thrust,ave}} \Delta t = <0, 1.84, 0> \text{ N s}
\]

\[
\bar{F}_{\text{thrust,ave}} = \frac{<0, 1.84, 0> \text{ N s}}{1.24 \text{ s}} = <0, 1.48, 0> \text{ N}
\]

The average thrust is sketched below.

![Thrust of an Estes Model Rocket Engine](image)

**Figure 2:** The average thrust and the area under the curve for the average thrust.

The impulse due to the average thrust is the area of the shaded rectangle. This area is equal to the area under the actual thrust vs. t curve.

(b) The average thrust is 1.48 which is much less than the maximum thrust of 8.6 N. The reason that the average is less than the maximum value is that the engine provides maximum thrust for a very small time
interval. Most of the burn time, the engine is exerting a smaller thrust, which decreases the overall average.

(c) According to the **Momentum Principle**, the net impulse causes a change in momentum

\[
\Delta \vec{p} = \vec{F}_{net} \Delta t
\]

If there are no other forces acting on the rocket, then \( \vec{F}_{net} = \vec{F}_{thrust} \) and

\[
\Delta \vec{p} = \vec{F}_{thrust} \Delta t
= <0, 1.84, 0> \text{ kg m/s}
\]

Since the thrust is in the +y direction, then the change in momentum of the rocket is also in the +y direction, assuming of course that the thrust is the only force acting on the rocket, which is an exceedingly simplistic assumption for a real model rocket which experiences forces by air and Earth as well.