**Question (1240001)**

**Momentum and position of a proton accelerated across charged plates**

In a mass spectrometer (an instrument used in chemistry to measure the mass and therefore identity of unknown atoms) positively charged ions are accelerated between two charged plates. Suppose that a proton is nearly at rest at the positively charged plate on the left. It is accelerated toward a negatively charged plate on the right, as shown below. The plates are 5 cm apart.

![Figure 1: A proton accelerated between charged plates.](image)

The electric force by the charged plates on the proton is $1.6 \times 10^{-13}$ N in the +x direction. This is the only significant force acting on the proton.

(a) If the initial velocity of the proton is zero, what is the velocity of the proton after a time interval of $5 \times 10^{-9}$ s?

(b) If you define the origin to be at the initial location of the proton, what is the position of the proton at $t = 5 \times 10^{-9}$ s?

**Solution**

(a) First, apply the **momentum principle**.

$$
\Delta \vec{p} = \vec{F}_{net} \Delta t
$$

Solve for an expression for the final momentum of the proton. This is called the update form of the momentum principle.

$$
\vec{p}_f - \vec{p}_i = \vec{F}_{net} \Delta t
$$

$$
\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t
$$
Sketch a free-body diagram showing the forces on the proton and find the net force. In this case, it’s quite simple because there is only one force acting on the proton and it is given to us as $<1.6 \times 10^{-13}, 0, 0> \text{ N}$.

\[
F_{\text{elec by plates on proton}} = \begin{array}{c}
\bullet \quad \rightarrow \\
F_{\text{net}}
\end{array}
\]

Figure 2: Free-body diagram for a proton accelerated between charged plates.

Solve for the final momentum of the proton.

\[
\vec{p}_f = <0, 0, 0> + (<1.6 \times 10^{-13}, 0, 0> \text{ N})(5 \times 10^{-9} \text{ s}) \\
= <8.0 \times 10^{-22}, 0, 0> \text{ kg s}
\]

Solve for the final velocity of the proton using the definition of momentum. Since the proton is not moving at close to the speed of light, then the nonrelativistic approximation for momentum can be used.

\[
\vec{p}_f \approx m \vec{v}_f \\
\vec{v}_f = \frac{\vec{p}_f}{m} \\
= \frac{<8.0 \times 10^{-22}, 0, 0> \text{ kg s}}{1.67 \times 10^{-27} \text{ kg}} \\
= <4.79 \times 10^5, 0, 0> \text{ m/s}
\]

Because the proton started at rest and the force on the proton is to the right, it’s not surprising to find that after a small time interval, the proton is traveling to the right with a greater speed. This agrees with the fact that it is accelerated to the right.

(b) To find the position of the proton at $t = 5 \times 10^{-9} \text{ s}$, use the definition of average velocity.

\[
\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \\
= \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \\
\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}} \Delta t
\]

But what is $\vec{v}_{\text{avg}}$? It must be approximated. Here are three simple ways of approximating $\vec{v}_{\text{avg}}$:~
\[\begin{align*}
(1) \quad \vec{v}_{avg} & \approx \vec{v}_i \\
(2) \quad \vec{v}_{avg} & \approx \vec{v}_f \\
(3) \quad \vec{v}_{avg} & \approx \frac{\vec{v}_f + \vec{v}_i}{2}
\end{align*}\]

Note that these aren't the only ways of approximating \(\vec{v}_{avg}\). They are the simplest. The arithmetic mean gives the most accurate result of the three methods so we'll choose that one.

\[
\vec{v}_{avg} = \frac{\vec{v}_f + \vec{v}_i}{2} = \frac{<4.79 \times 10^5, 0, 0>}{2} = <2.40 \times 10^5, 0, 0> \text{ m/s}
\]

Now, we can use the update form of the definition of velocity to calculate the final position of the proton. The initial position of the proton is at the origin, and the final position after \(t = 5 \times 10^{-9}\) s is

\[
\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = 0 + (<2.40 \times 10^5, 0, 0> \text{ m/s})(5 \times 10^{-9} \text{ s}) = <0.0012, 0, 0> \text{ m}
\]

Note that the proton moved only 0.0012 m which is 1.2 mm. That's reasonable considering that the time interval is very small and the distance between the plates is 5 cm.

So, do you think that you can calculate how fast the proton is moving and where it is located after the next time interval of \(5 \times 10^{-9}\) s? Just repeat the procedure and assume that the net force on the proton is constant.