Question (2g30002)

An electron deflected by charged plates.

An electron enters a region of uniform electric field between two closely spaced, oppositely charged plates as shown below with an initial speed of $1.0 \times 10^6$ m/s. Upon exiting the region, it has been deflected upward. The horizontal displacement of the electron through the plates is 5 cm, and the plates are separated a distance 5 mm.

![Figure 1: An electron deflected by oppositely charged plates.](image)

(a) Sketch the electric field between the plates.
(b) Which plate is positively charged and which plate is negatively charged?
(c) Which plate is at a higher electric potential $V$?
(d) Sketch the path of the electron as it travels through the plates.
(e) If the vertical deflection of the electron is 1 mm, what is the potential difference across the plates?

Solution

(a) Because the plates are closely spaced, the electric field between the plates is nearly uniform and the electric force on the electron will be constant throughout the region between the plates. The electron is deflected upward; therefore, there must be an upward force on the electron. Since $\vec{F} = q\vec{E}$ and $q$ is negative, the electric field must point downward, as shown below.

![Figure 2: Electric field and force on an electron between plates.](image)
(b) The electric field points away from the positively charged plate and toward the negatively charged plate; therefore, the top plate must be positively charged and the bottom plate negatively charged.

![Electric field diagram](image.png)

Figure 3: Top plate is positively charged and the bottom plate is negatively charged.

(c) Because the electric field between the plates is constant, the electric potential varies linearly with $y$. Electric field points from high potential to low potential. Therefore, the top plate is at a higher potential than the lower plate.

(d) Because the force on the electron is in the $+y$ direction, the electron’s $x$-velocity will be constant and its $y$-velocity will increase in the $+y$ direction. As a result, the electron’s path will be a parabola, much like projectile motion.

![Path of electron](image.png)

Figure 4: The path of an electron traveling between the plates is a parabola.

(e) The potential difference between the plates for a constant electric field is

$$\Delta V = -\vec{E} \cdot \Delta \vec{L}$$

Since the electric field points in the $-y$ direction only,

$$\Delta V = -(E_x \Delta x + E_y \Delta y + E_z \Delta z) = -(E_y \Delta y)$$

$$|\Delta V| = |\vec{E}|s$$

where $s$ is the plate separation and $|\vec{E}|$ is the magnitude of the electric field between the plates. Thus, we need to calculate the electric field, using $\vec{F} = q\vec{E}$ and we can use the motion of the electron and The Momentum Principle to get $\vec{F}$. So, the next step is to calculate $\vec{F}$ using The Momentum Principle.
\[ \vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} \]

Because the net force on the electron in the x-direction is zero, the \( p_x \) and \( v_x \) is constant. Thus, \( v_x = 1.0 \times 10^6 \) m/s. The time interval during which the electron travels across the region a displacement \( \Delta x \) is

\[ v_x = \frac{\Delta x}{\Delta t} \]

\[ \Delta t = \frac{\Delta x}{v_x} \]
\[ = \frac{0.05 \text{ m}}{1.0 \times 10^6 \text{ m/s}} \]
\[ = 5 \times 10^{-8} \text{ s} \]

Now, apply Newton's second law in the y-direction.

\[ F_y = \frac{\Delta p_y}{\Delta t} \]
\[ = \frac{p_{y,f} - p_{y,i}}{\Delta t} \]
\[ = \frac{p_{y,f} - 0}{\Delta t} \]
\[ = \frac{p_{y,f}}{\Delta t} \]
\[ = \frac{m v_{y,f}}{\Delta t} \]

The final y-velocity must be obtained by considering the vertical deflection \( \Delta y \). The average y-velocity is

\[ v_{y,\text{ave}} = \frac{v_{y,i} + v_{y,f}}{2} = \frac{\Delta y}{\Delta t} \]

Solving for the final y-velocity gives

\[ \frac{v_{y,i} + v_{y,f}}{2} = \frac{\Delta y}{\Delta t} \]
\[ 0 + v_{y,f} = \frac{0.001 \text{ m}}{5 \times 10^{-8} \text{ s}} \]
\[ v_{y,f} = \frac{2(0.001) \text{ m}}{5 \times 10^{-8} \text{ s}} \]
\[ v_{y,f} = 4.0 \times 10^4 \text{ m/s} \]
Thus, the net force in the y-direction on the electron is

\[ F_y = \frac{mv_{y,f}}{\Delta t} = \frac{(9.11 \times 10^{-31} \text{ kg})(4.0 \times 10^4 \text{ m/s})}{5 \times 10^{-8} \text{ s}} = 7.29 \times 10^{-19} \text{ N} \]

The force of the electric field on the electron is the ONLY force acting on the electron. (The gravitational force on the electron is negligible.) Thus, the electric field is

\[ E_y = \frac{F_y}{q} = \frac{7.29 \times 10^{-19} \text{ N}}{-1.6 \times 10^{-19} \text{ C}} \]

\[ |E| = 4.6 \text{ N/C} \]

The potential difference across the plates is

\[ |\Delta V| = |E|s = (4.6 \text{ N/C})(0.005 \text{ m}) = 0.023 \text{ V} \]