Question (1150004)

Displacement; direction cosines

A girl walks from the position \( \vec{r}_i = (2, -4, 3) \) m to the position \( \vec{r}_f = (-8, 5, 0) \) m.

(a) How far is the girl from the origin when she’s at \( \vec{r}_f \)?

(b) What is her displacement?

(c) What is the magnitude and direction of her displacement?

(d) What angle does her displacement make with the x-axis?

(e) What angle does her displacement make with the y-axis?

(f) What angle does her displacement make with the z-axis?

Solution

Sketch a picture in order to visualize the situation. Though the vectors are 3-D, it’s usually easiest on paper to sketch just two of the dimensions.

![Figure 1: Projections of a girl’s initial position, final position, and displacement, in the x-y plane.](image)

(a) To get “how far” she is from the origin, the magnitude of the position vector must be calculated.

\[
|\vec{r}_f| = \sqrt{(-8)^2 + 5^2 + 0^2} \text{ m} \\
= \sqrt{89} \text{ m} \\
= 9.4 \text{ m}
\]
(b) Displacement is the change of position.

\[
\Delta \vec{r} = \vec{r}_f - \vec{r}_i = \langle -8, 5, 0 \rangle \text{ m} - \langle -2, -4, 3 \rangle \text{ m} = \langle -10, 9, -3 \rangle \text{ m}
\]

(c) Magnitude and direction are

\[
|\Delta \vec{r}| = \sqrt{(-10)^2 + 9^2 + (-3)^2} \text{ m} = \sqrt{190} \text{ m} \\
= 13.8 \approx 14 \text{ m}
\]

\[
\hat{\Delta \vec{r}} = \frac{\Delta \vec{r}}{|\Delta \vec{r}|} = \langle -0.72, 0.65, -0.22 \rangle
\]

(d) Use the unit vector to calculate the direction cosines. For \( \theta_x \):

\[
\cos(\theta_x) = \frac{\Delta \vec{r}_x}{|\Delta \vec{r}|} = \Delta r_x \\
\cos(\theta_x) = -0.72
\]

Therefore,

\[
\theta_x = \cos^{-1}(-0.72) \\
= 136^\circ \approx 140^\circ
\]

Compare this to the picture, though the picture is only showing a two-dimensional projection of the vector.

(e) For \( \theta_y \):

\[
\cos(\theta_y) = \frac{\Delta \vec{r}_y}{|\Delta \vec{r}|} = \Delta r_y \\
\cos(\theta_y) = 0.65
\]

Therefore,
\[
\theta_y = \cos^{-1}(0.65) \\
= 49^\circ
\]

Compare this to the picture. 49 degrees (counterclockwise) from the +y axis will be in the second quadrant. The picture of the displacement vector confirms this calculation.

In the picture shown below, the displacement vector has been moved to the origin of the coordinate system, and angles with respect to the +x axis and +y axis are shown. The z-axis and z-component of the vector are not shown in this 2-D view, thus the picture is not entirely accurate.

![Figure 2: Angles between the displacement vector in the x-y plane and the +x axis and +y axis respectively.](image)

(f) For \(\theta_z\):

\[
\cos(\theta_z) = \frac{\Delta \vec{r}_z}{|\Delta \vec{r}|} = \frac{\Delta r_z}{|\Delta \vec{r}|}
\]

\[
\cos(\theta_z) = -0.22
\]

Therefore,

\[
\theta_z = \cos^{-1}(-0.22) \\
= 103^\circ \approx 100^\circ
\]

The +z axis points out of the page in this 2-D view of the x-y plane. Our 3-D vector has a z-component of -3 m. Therefore, it points into the page; therefore, it makes sense that the angle with the +z axis is greater than 90 degrees, since less than 90 degrees would point out of the page and 90 degrees would be the x-y plane (and the z-component of the vector would then have to be zero).