Physics 221

Midterm – Fall 2004, Form: A

Name: ____________________

Date: ____________________

Content covers chapters 2–4.

Section 1. Vectors

Questions 1–4: In a new movie, FBI agents track an airplane using radar. The airplane moves from the position $< 30, 3, -10 >$ km to $< 20, 2.5, 20 >$ km.

1. Sketch the initial position, final position, and displacement vectors in the x-z plane. (i.e. just draw the x and z components with +x to the right and +z upward)

2. What is the displacement of the airplane?

3. What is the magnitude of the displacement of the airplane?

4. What is the direction of the momentum of the airplane during this displacement? (assuming constant momentum) Express this as a unit vector.

Section 2. Momentum Principle

Questions 5–10:

5. You are a practicing ornithologist studying migratory patterns of birds by analyzing video of a flock of birds, shot from ground level. You analyze the motion of the lead bird. The time interval between two frames of video is 0.0333 s. Data for 4 successive frames of video is shown below in Table 1. The x-z plane is defined as the plane of the video with +x to the right and +z upward. The +y direction is the elevation of the bird above the ground with y=0 defined to be their elevation at the first frame of video.

Frame Number	time (s)	position (m)
1	0	$< -4.37354, 0, 0.228013 >$
2	0.0333	$< -3.74708, 0, 0.456027 >$
3	0.0667	$< -3.12061, 0, 0.68404 >$
4	0.1	$< -2.49415, 0, 0.912054 >$

Table 1: Position and Time Data for the Lead Bird.

An average-sized bird of this species has a mass of about 1.5 kg. What is the momentum of the lead bird from frame 1 to frame 2?

6. Is the momentum of the bird constant? Explain your answer using both words and calculations.

7. What do you predict that the position of the bird will be in frame 5?

8. While analyzing the video, you notice that a bird of prey speeds up in order to catch up to the flock. As a typical sized bird of prey, its mass is about 3 kg. After making careful measurements, you find that it speeds up from a speed of 15 m/s to a speed of 25 m/s during a time interval of 15 s. Define its direction of flight as the +x direction and the upward direction (from the ground) as the +y direction.

   Assuming a constant net force, what is the net force on the bird during this interval.

9. What forces act on the bird of prey? Sketch vectors for the forces, label them, and sketch the net force.

10. What is the force of air on the bird? Start from a fundamental principle.

Section 3. Momentum principle applied to a system with a spring

11. A new exhibit at a children's museum teaches children about the importance of sports and exercise for good health. A stiff spring with one end attached to the wall allows children to test their strength. A child pulls horizontally on a handle attached to the other end of the spring and a digital readout on the wall displays the maximum magnitude of the force, in newtons, that the child exerts on the spring.

    Little Billy pulls on the handle and stretches the spring a maximum amount of 0.13 m. The readout says 6.5 N. Little Susie pulls on the handle and stretches it a maximum amount of 0.2 m. What force (maximum force) did Susie exert on the handle of the spring?

Section 4. Momentum principle applied to uniform circular motion

12. You are a little worried about the safety of the Bowman Beast, a new ride at the local amusement park meant to test human ability to withstand high net force. It's basically a human centrifuge that rotates in a vertical circle. You are fairly certain that the typical person wouldn't enjoy any ride where the net force on the person is greater than 3 times their weight.

    Suppose a person of mass 68 kg rides this amusement park ride. The radius of the circle is 15 m and its period is 5.0 s. Calculate the magnitude of the net force on a rider and decide whether the typical person would enjoy this ride.

13. When the rider is at the bottom, what is the force of the seat (including harness) on the rider? Start with a fundamental principle.

Section 5. Energy Principle

14. In outer space, a piece of space junk whose mass is 60 kg is subject to a constant net force $< 13, -9, 12 >$ N. When the space junk is at location $< 10, 6, -4 >$ m, its speed is 0.8 m/s. When the space junk has moved to location $< 13, 8, -2 >$ m what is its speed?

15. You blast off from Mars, and you turn off the rockets when you are 3500 km (3.5×10^{6} m) from the center of Mars, well above its thin atmosphere and headed away from the planet. You intend to leave Mars for good, and by the time you get very far away you want to be coasting at a speed of 1800 m/s. Mars has a mass of 6.4×10^{23} kg.

    Calculate the speed you must have when you are 3500 km (3.5×10^{6} m) from the center of Mars in order that your speed when you are very far from Mars is 1800 m/s. Explain carefully and completely, starting from a fundamental principle.

Questions 16–18: A bungee jumper in West Virginia "jumps" off a tall bridge, starting from rest. The unstretched length of the bungee cord is 40 m and its stiffness is 10 N/m. The jumper's mass is 90 kg.

16. During the first 40 m of falling, the bungee cord is unstretched during the entire interval. What is speed of the jumper after falling 40 m? Sketch a picture and apply a fundamental principle.

17. When the bungee jumper reaches her lowest point, how much is the bungee cord stretched? Sketch a picture and apply a fundamental principle.

18. What must be the height of the bridge so that the jumper does not hit the ground? Note, a picture helps to answer this question!

Answer Key for Exam A

Section 1. Vectors

Questions 1–4: In a new movie, FBI agents track an airplane using radar. The airplane moves from the position $< 30, 3, -10 >$ km to $< 20, 2.5, 20 >$ km.

1. Sketch the initial position, final position, and displacement vectors in the x-z plane. (i.e. just draw the x and z components with +x to the right and +z upward)

   $\vec{r}_i$ is down and to the right.
   $\vec{r}_f$ is up and to the right.

2. What is the displacement of the airplane?

   $\Delta\vec{r} = \vec{r}_f - \vec{r}_i =< -10, -0.5, 30 >$ km

3. What is the magnitude of the displacement of the airplane?

   31.6 km

4. What is the direction of the momentum of the airplane during this displacement? (assuming constant momentum) Express this as a unit vector.

   $\hat{p} =< -0.316, -0.00158, 0.949 >$

Section 2. Momentum Principle

Questions 5–10:

5. You are a practicing ornithologist studying migratory patterns of birds by analyzing video of a flock of birds, shot from ground level. You analyze the motion of the lead bird. The time interval between two frames of video is 0.0333 s. Data for 4 successive frames of video is shown below in Table 1. The x-z plane is defined as the plane of the video with +x to the right and +z upward. The +y direction is the elevation of the bird above the ground with y=0 defined to be their elevation at the first frame of video.

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3	0.0667	$< -3.12061, 0, 0.68404 >$
4	0.1	$< -2.49415, 0, 0.912054 >$

Table 2: Position and Time Data for the Lead Bird.

An average-sized bird of this species has a mass of about 1.5 kg. What is the momentum of the lead bird from frame 1 to frame 2?

$\vec{p} = 30 < 0.940, 0, 0.342 >=< 28.2, 0, 10.3 >$ kg m/s

6. Is the momentum of the bird constant? Explain your answer using both words and calculations.

The displacement between any two successive frames of video is the same. Therefore, the velocity and momentum are constant.

7. What do you predict that the position of the bird will be in frame 5?

   $\vec{r_f} =< -1.87, 0, 1.14 >$ m

8. While analyzing the video, you notice that a bird of prey speeds up in order to catch up to the flock. As a typical sized bird of prey, its mass is about 3 kg. After making careful measurements, you find that it speeds up from a speed of 15 m/s to a speed of 25 m/s during a time interval of 15 s. Define its direction of flight as the +x direction and the upward direction (from the ground) as the +y direction.

   Assuming a constant net force, what is the net force on the bird during this interval.

   $\vec{F}_{net} = 3 < 10, 0, 0 > /15 =< 2, 0, 0 >$ N

9. What forces act on the bird of prey? Sketch vectors for the forces, label them, and sketch the net force.

   gravitational force downward, force of air on the bird upward and to the right, net force to the right.

10. What is the force of air on the bird? Start from a fundamental principle.

$\vec{F}_{net} = 3 < 10, 0, 0 > /15 =< 2, 0, 0 >$ N
$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{air}$
$\vec{F}_{air} =< 2, 0, 0 > - < 0, -3(9.8), 0 >=< 2, 29.4, 0 >$ N

Section 3. Momentum principle applied to a system with a spring

11. A new exhibit at a children's museum teaches children about the importance of sports and exercise for good health. A stiff spring with one end attached to the wall allows children to test their strength. A child pulls horizontally on a handle attached to the other end of the spring and a digital readout on the wall displays the maximum magnitude of the force, in newtons, that the child exerts on the spring.

    Little Billy pulls on the handle and stretches the spring a maximum amount of 0.13 m. The readout says 6.5 N. Little Susie pulls on the handle and stretches it a maximum amount of 0.2 m. What force (maximum force) did Susie exert on the handle of the spring?

    $|\vec{F}_{spring}| = ks$
    $k = 6.5/0.13 = 50$ N/m
    $|\vec{F}_{spring}| = 50 * 0.2 = 10$ N

Section 4. Momentum principle applied to uniform circular motion

12. You are a little worried about the safety of the Bowman Beast, a new ride at the local amusement park meant to test human ability to withstand high net force. It's basically a human centrifuge that rotates in a vertical circle. You are fairly certain that the typical person wouldn't enjoy any ride where the net force on the person is greater than 3 times their weight.

    Suppose a person of mass 68 kg rides this amusement park ride. The radius of the circle is 15 m and its period is 5.0 s. Calculate the magnitude of the net force on a rider and decide whether the typical person would enjoy this ride.

    $v = 2\pi r/T = 18.85$ m/s
    $|\vec{F}_{net}| = mv^2/r = 1611$ N; $|\vec{F}_{grav}| = mg = 666$ N
    The net force is less than 3 x 666 N, so the rider should, theoretically, enjoy the ride.

13. When the rider is at the bottom, what is the force of the seat (including harness) on the rider? Start with a fundamental principle.

    The net force on the rider is toward the center (upward). The gravitational force of Earth on the rider is downward. The only other force on the rider is the force of the seat which is upward.

    $\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{seat}$
    $\vec{F}_{seat} = \vec{F}_{net} - \vec{F}_{grav}$
    $\vec{F}_{seat} =< 0, 1611, 0 > - < 0, -666, 0 >=< 0, 2277, 0 >$ N

Section 5. Energy Principle

14. In outer space, a piece of space junk whose mass is 60 kg is subject to a constant net force $< 13, -9, 12 >$ N. When the space junk is at location $< 10, 6, -4 >$ m, its speed is 0.8 m/s. When the space junk has moved to location $< 13, 8, -2 >$ m what is its speed?

    $W = \vec{F}_{net} \cdot \Delta\vec{r} = 45$ J
    $W = \Delta E = \Delta K$, system is the space junk
    . $v_f = 1.46$ m/s

15. You blast off from Mars, and you turn off the rockets when you are 3500 km (3.5×10^6 m) from the center of Mars, well above its thin atmosphere and headed away from the planet. You intend to leave Mars for good, and by the time you get very far away you want to be coasting at a speed of 1800 m/s. Mars has a mass of 6.4×10^{23} kg.

    Calculate the speed you must have when you are 3500 km (3.5×10^6 m) from the center of Mars in order that your speed when you are very far from Mars is 1800 m/s. Explain carefully and completely, starting from a fundamental principle.

    $W = \Delta E$, system is the space ship and planet
    $W = 0$ since no external forces do work on the system.
    $\Delta K + \Delta U = 0$
    $v_i = 5270$ m/s

Questions 16–18: A bungee jumper in West Virginia "jumps" off a tall bridge, starting from rest. The unstretched length of the bungee cord is 40 m and its stiffness is 10 N/m. The jumper's mass is 90 kg.

16. During the first 40 m of falling, the bungee cord is unstretched during the entire interval. What is speed of the jumper after falling 40 m? Sketch a picture and apply a fundamental principle.

    $mgy = 1/2mv^2$
    $v = \sqrt{2gy} = 28$ m/s

17. When the bungee jumper reaches her lowest point, how much is the bungee cord stretched? Sketch a picture and apply a fundamental principle.

    $1/2mv_i^2 + mgy_i = 1/2ks_f^2$ where $y_i = s_f$
    $s = 210$ m

18. What must be the height of the bridge so that the jumper does not hit the ground? Note, a picture helps to answer this question!

    The bridge must be at least $210 + 40$ m high (250 m).