Questions 1–3: State your answers to the following questions mathematically. Your answers must be exactly correct and all vector quantities should be written with an arrow above the variable.

1. What is the definition of velocity (Hint: it includes displacement and time)?

2. What is the definition of momentum that is approximately correct for an object moving at speeds much less than the speed of light?

3. What is the momentum principle?

Questions 4–7: An object of mass \( m \) travels around a circle of radius \( r \) with a constant speed \( |\vec{v}| \). State your answers to the following questions using mathematical equations.

4. What is the speed of the object in terms of the radius of the circle \( r \) and the period \( T \)?

5. What is the magnitude of the net force on the object at any instant of time, assuming that the speed of the object is much less than the speed of light?
6. If the object in question is a satellite of mass $m$ in a circular orbit around a much more massive planet of mass $M$ such that the net force on the satellite is equal to the gravitational force of the planet on the satellite, what is the speed of the satellite?

7. For the satellite in the previous question, what is its period in terms of the radius of its orbit and the mass of the planet?

8. You run at a speed of 12 m/s around a circular portion of a track of radius 12 m. What is the magnitude of the force of the track on you. Assume your mass is about 60 kg.
9. A satellite orbiting the moon is located at the position shown. If the Earth is the origin, and the moon is at \( <3.33 \times 10^8, 1.92 \times 10^9, 0> \) and the satellite is at \( <3.39 \times 10^8, 1.89 \times 10^9, 0> \). What is the gravitational force of the moon on the satellite? (Note: express this as a vector and sketch it on the diagram.) The mass of the moon is \( M_{\text{moon}} = 7.35 \times 10^{22} \) kg and the mass of the satellite is \( 1 \times 10^5 \) kg.

![Figure 1:](image-url)
10. You push a 40-kg box on a floor as shown in the image below.

![Figure 2:](image)

You push with a force of 100 N at an angle, with respect to the horizontal, of 15 °. If the box moves in the +x direction with a constant speed, what is the force of the floor on the box?

11. For the previous question, what is the frictional force on the box?
Answer Key for Exam A

1. 

\[ \vec{v} = \frac{\Delta \vec{r}}{\Delta t} \]

2. 

\[ \vec{p} = m \vec{v} \]

3. 

\[ \vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} \]

4. 

\[ |\vec{v}| = 2\pi r/T \]

5. 

\[ |\vec{F}_{net}| = m|\vec{v}|^2/r \]

6. 

\[ |\vec{v}| = \sqrt{GM/r} \]

7. 

\[ T^2 = \frac{4\pi^2}{GM^3} \]

8. Sketch a picture of the circle and the runner. I’ll place the runner at a location to the right of the center of the circle so that the rate of change of her momentum is to the left, toward the center of the circle. Define the +x direction to the right and the +y direction perpendicular to the track. Neglect air resistance. Define the system to be the runner.

\[ \vec{F}_{net} = \frac{d\vec{p}}{dt} \]

\[ \vec{F}_{grav} + \vec{F}_{track} = -mv^2/r, 0, 0 \]

\[ \vec{F}_{track} = -mv^2/r, 0, 0 > - < 0, -mg, 0 > \]

\[ \vec{F}_{track} = -mv^2/r, -mg, 0 > = < 720, 588, 0 > \text{ N} \]

\[ |\vec{F}_{track}| = 930 \text{ N} \]
9. To make things a bit easier, I’ll define the vector $\vec{r}$ to be toward Moon.

$$\vec{r} = \vec{r}_{moon} - \vec{r}_{sat} = \langle -0.06, 0.03, 0 \rangle \times 10^8 \text{ m}$$

$$\hat{r} = \langle -0.90, 0.45, 0 \rangle$$

$$|\vec{r}| = 6.7 \times 10^6 \text{ m}$$

$$\vec{F}_{grav} = \frac{GMm}{r^2} \hat{r} = 1.1 \times 10^4 \langle -0.90, 0.45, 0 \rangle \text{ N}$$

Note that I wrote the gravitational force in terms of its magnitude times its direction. It’s easy enough to just multiply it out.

10. The system is the box. Forces on the box are due to Earth, the floor, and you (i.e. your hands). The box’s velocity is constant, so it is in equilibrium.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = 0$$

$$\vec{F}_{grav} + \vec{F}_{floor} + \vec{F}_{me} = 0$$

$$\vec{F}_{floor} = -\vec{F}_{grav} - \vec{F}_{me}$$

$$\vec{F}_{floor} = -\langle 0, -mg, 0 \rangle - \langle F_{me} \cos(15), -F_{me} \cos(90 - 15), 0 \rangle$$

$$\vec{F}_{floor} = \langle -96.6, 418, 0 \rangle$$

11. The frictional force of the floor on the box is just the component of the force of the floor on the box that is parallel to the floor. That’s the x-component, 96.6 N, in the -x direction (or -96 N).