Physics 221

Quiz 6 — chapters 7-9, Form: A

Name: ______________________

Date: ______________________

An apparatus consists of three balls each of mass $M = 0.6$ kg mounted on the ends of low-mass rods of length $R = 0.24$ m. The whole apparatus rotates clockwise in the plane of the page around its center at a constant rate, going around once every 2.8 seconds. The center of mass of the apparatus moves downward with a constant speed of 0.5 m/s.

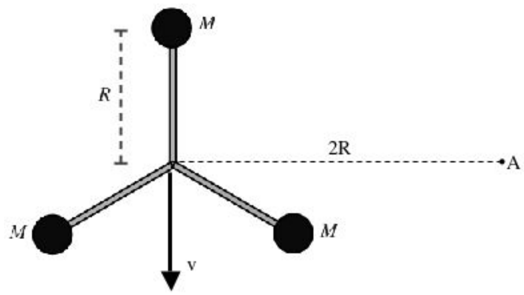

Figure 1:

1. What is the momentum of the system?

2. What is the angular momentum of the system with respect to point A? (Hint: the total angular momentum can be expressed as translational angular momentum plus rotational angular momentum. Write your answer as a vector!)

3. What is the kinetic energy of the apparatus? (Hint: the total kinetic energy can be expressed as translational kinetic energy plus rotational kinetic energy.)

A puck moving on ice with a speed 0.5 m/s collides with a puck that is attached to another puck via a very lightweight rigid rod, as shown in the figure below. Let's refer to the attached pucks as a "rotor." After the collision, the incoming puck rebounds backward with a speed of 0.1 m/s and the rotor moves to the right and rotates clockwise. Kinetic energy is not conserved during the collision. All pucks have the same mass of 0.1 kg and the length of the rod is 0.8 m.

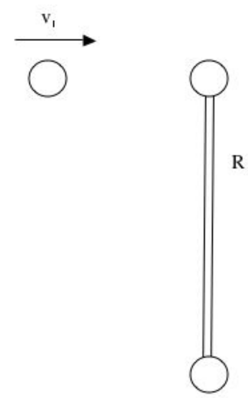

Figure 2:

4. Using the momentum principle, calculate the velocity of the center of mass of the rotor after the collision. (Note: if the system is defined to be the set of three pucks, the net force on the system during the collision is zero.)

5. Using the angular momentum principle, what is the angular velocity vector for the rotor after the collision? (Note: if the system is defined to be the set of three pucks, the net torque on the system during the collision is zero.)

6. What are all of the possible values of the orbital angular momentum of an electron in a hydrogen-like atom that is in the 3rd excited energy state? Give your answer in terms of $\hbar$.

7. For the angular momentum vector corresponding to the $l = 3$ quantum number, what are the possible orientations of the orbital angular momentum vector with respect to the z-axis? Calculate each allowed L_z in terms of $\hbar$ and sketch a picture of the angular momentum vector $\vec{L}$ for each case.

8. What are all the possible orientations of the spin angular momentum of an electron in a hydrogen-like atom? Calculate the possible values of the z-component of the spin angular momentum in units of $\hbar$ and sketch the spin angular momentum vector $\vec{S}$ for each allowed orientation. Does this depend on the energy state (n) of the atom?

A box contains machinery that can rotate. The total mass of the box plus machinery is 2 kg. A string wound around the machinery comes out through a small hole in the top of the box. Initially the box sits on the ground, and the machinery inside the box is not rotating. Then you pull upwards on the string with a constant force of magnitude 30 N. At an instant when you have pulled a length of string 1.0 m out of the box, the box has risen a height 0.2 m.[1]

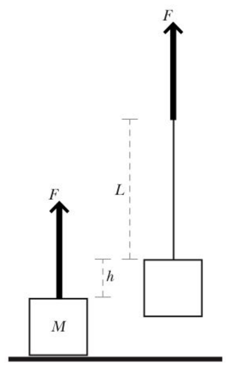

Figure 3:

9. Consider the point particle system and calculate the speed of the box at this instant.

10. Consider the real system and calculate the rotational kinetic energy of the machinery inside the box at this instant.

[1]authored by Bruce Sherwood, NCSU

Answer Key for Exam A

1. $\vec{p} = m\vec{v} = 3(0.6) < 0, -0.5, 0 > = < 0, -0.9, 0 >$ kg m/s

2. $\vec{L} = \vec{L}_{trans} + \vec{L}_{rot}$

   $\vec{L}_{trans} = \vec{r}_{cm} \times \vec{p}_{cm} = < 0, 0, 2Rp_{cm} > = < 0, 0, 0.432 >$ kg m^2/s

   $\vec{L}_{rot} = I\vec{\omega} = (3mR^2) < 0, 0, -\frac{2\pi}{T} > = < 0, 0, -0.232 >$ kg m^2/s

   $\vec{L} = < 0, 0, 0.2 >$ kg m^2/s

3. $K = K_{trans} + K_{rot}$

   $K_{trans} = \frac{1}{2}(3m)v_{cm}^2 = 0.225$ J

   $K_{rot} = \frac{1}{2}(3mR^2)(\frac{2\pi}{T})^2 = 0.260$ J

   $K = 0.485$ J

4. $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t} = 0$ so $\vec{p}_f = \vec{p}_i$

   $\vec{p}_i = m\vec{v}_{1i}$
   $\vec{p}_f = m\vec{v}_{1f} + (2m)\vec{v}_{rf}$

   $\vec{p}_f = \vec{p}_i$
   $m\vec{v}_{1i} = m\vec{v}_{1f} + (2m)\vec{v}_{rf}$
   $\vec{v}_{rf} = \frac{\vec{v}_{1i} - \vec{v}_{1f}}{2} = \frac{<0.5,0,0> - <-0.1,0,0>}{2} = < 0.3, 0, 0 >$ m/s

5. $\vec{\tau}_{net} = \frac{\Delta\vec{L}}{\Delta t} = 0$ so $\vec{L}_f = \vec{L}_i$

   $\vec{L}_i = \vec{r_i} \times \vec{p_i} = < 0, 0, -Rp_{1i} >$
   $\vec{L}_f = \vec{L}_{1f} + \vec{L}_{rf}\vec{r_f} \times \vec{p_f} + I\vec{\omega} = < 0, 0, Rp_{1f} > + < 0, 0, -(2mR^2)\omega_f >$

   $\vec{L}_i = \vec{L}_f$
   $< 0, 0, -Rp_{1i} > = < 0, 0, Rp_{1f} > + < 0, 0, -(2mR^2)\omega_f >$
   $-Rp_{1i} = Rp_{1f} + -2mR^2\omega_f$
   $\omega_f = \frac{-Rp_{1i} - Rp_{1f}}{2mR^2} = \frac{-v_{1i} - v_{1f}}{2R} = -0.75$ rad/s

   $\vec{\omega}_f = < 0, 0, -0.75 >$ rad/s

6. $n = 4$ so $l = 0, 1, 2, 3$

   $L\sqrt{l(l+1)}\hbar$

   $l = 0 \qquad L = 0$
   $l = 1 \qquad L = \sqrt{2}\hbar$

$l = 2 \qquad L = \sqrt{6}\hbar$
$l = 3 \qquad L = \sqrt{12}\hbar$

7. $m_l = -l, -l+1, ..., l$ and $L_z = m_l\hbar$

   $m_l = -3 \qquad L_z = -3\hbar$
   $m_l = -2 \qquad L_z = -2\hbar$
   $m_l = -1 \qquad L_z = -1\hbar$
   $m_l = 0 \qquad L_z = 0$
   $m_l = 1 \qquad L_z = 1\hbar$
   $m_l = 2 \qquad L_z = 2\hbar$
   $m_l = 3 \qquad L_z = 3\hbar$

   (no picture included on this key)

8. $S = sqrts(s+1)\hbar$ where $s = \frac{1}{2} for an electron$
   $S = \frac{\sqrt{3}}{4}\hbar$
   $S_z = m_s\hbar \qquad m_s = -\frac{1}{2} or \frac{1}{2}$
   $m_s = -\frac{1}{2} \qquad S_z = -\frac{1}{2}\hbar$
   $m_s = \frac{1}{2} \qquad S_z = \frac{1}{2}\hbar$

   (no picture included with this key)

9. $\Delta K_{trans} = \vec{F}_{net} \cdot \Delta\vec{r}_{cm}$
   $\frac{1}{2}mv_{cm,f}^2 - \frac{1}{2}mv_{cm,i}^2 = (F - mg)h$

   $v_{cm,f} = sqrt\frac{2(F-mg)h}{m} = 1.44$ m/s

10. $\Delta E = W + Q$ and the system is defined to be the earth and box
    $Q = 0$ since it is thermally isolated
    $\Delta K_{trans} + \Delta K_{rot} + \Delta U_{grav} = F(L + h)$
    $(F - mg)h + \Delta K_{rot} + mgh = F(L + h)$
    $\Delta K_{rot} = FL = 30$ J