CH 18-3 – Charging and Discharging Capacitor

Important Ideas

Charging Capacitor

- For a charging capacitor, current flows and charge builds up on each plate. Eventually the electric field pushing the current is balanced by the opposite electric field due to the charge on each plate, and the net electric field is zero. Then the current will be zero. The charge on a plate is zero at $t = 0$ and increases until it reaches a maximum value. $Q(t)$ is

  $$Q = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

  The voltage across the capacitor is $\Delta V = \frac{Q}{C}$. Thus,

  $$\Delta V_C = \Delta V_{C,max}\left(1 - e^{-\frac{t}{RC}}\right)$$

  The current starts out as a maximum and decreases until it goes to zero.

  $$I = I_{max}e^{-\frac{t}{RC}}$$

  The voltage across the resistor is $\Delta V = IR$. Thus,

  $$\Delta V_R = \Delta V_{R,max}e^{-\frac{t}{RC}}$$

  The maximum voltage across the capacitor and resistor is the emf of the battery. Applying KVL to the circuit,

  $$\Delta V_{bat} = \Delta V_C + \Delta V_R$$

  Thus, as the capacitor charges, ΔV_C increases until it is equal to the voltage across the battery, and ΔV_R decreases until it is zero.

Discharging Capacitor

- For a discharging capacitor, current flows from the $+Q$ plate to the $-Q$ plate. The electric potential energy stored in the capacitor is dissipated by the resistor (i.e. converted to thermal energy). The electric field pushing the current is due to the charge on the capacitor plates. As the charge flows from one plate to the other, the charge on each plate decreases to zero, and the current decreases to zero. The charge on each plate is

  $$Q = Q_{max}\left(e^{-\frac{t}{RC}}\right)$$

  The voltage across the capacitor is $\Delta V = \frac{Q}{C}$. Thus,

$$\Delta V_C = \Delta V_{C,max}\left(e^{-\frac{t}{RC}}\right)$$

The current starts out as a maximum and decreases until it goes to zero.

$$I = I_{max}e^{-\frac{t}{RC}}$$

The voltage across the resistor is $\Delta V = IR$. Thus,

$$\Delta V_R = \Delta V_{R,max}e^{-\frac{t}{RC}}$$

The maximum voltage across the capacitor is its initial voltage. Applying KVL to the circuit,

$$|\Delta V_C| = |\Delta V_R|$$

since the capacitor is simply discharging across the resistor. (There is no battery.)

- The energy stored in a capacitor is $U = \frac{1}{2}C(\Delta V)^2$.
- The time constant is $\tau = RC$ and is equal to the time for a quantity to decrease to 37% of its initial value or increase to 63% of its initial value, depending on whether it is increasing or decreasing.

Examples

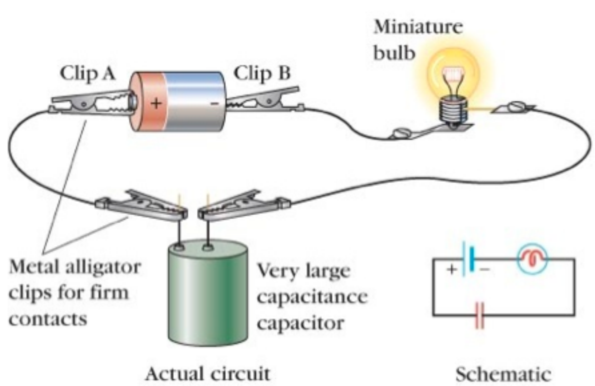
1. You discharge a capacitor using a resistor R. You then charge it again and repeat the experiment, but this time connecting a single wire across the capacitor (i.e. a short circuit).

   (a) It will take more time to fully discharge the capacitor.

   (b) It will take less time to fully discharge the capacitor.

   (c) It will take the same time to fully discharge the capacitor.

2. An RC circuit is composed of a battery, capacitor, and resistor shown below. Suppose that the capacitor is initially uncharged when the last wire is connected. Which plate will be positively charged after some time interval t?



3. An RC circuit is composed a 3-V battery, a 100 μF capacitor and a 100 kΩ resistor. At $t = 0$, the capacitor is uncharged when the switch is closed and current begins to flow.

   (a) What is the time constant of this circuit?

   (b) At $t = 0$

      i. How much charge is on each plate of the capacitor?
      ii. What is the voltage across the capacitor?
      iii. What is the voltage across the resistor?
      iv. What is the current?

   (c) At $t = \tau$

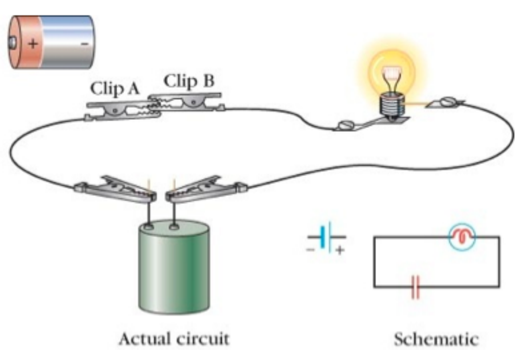
      i. How much charge is on each plate of the capacitor?
      ii. What is the voltage across the capacitor?
      iii. What is the voltage across the resistor?
      iv. What is the current?

   (d) After a very long time (perhaps a few minutes)

      i. How much charge is on each plate of the capacitor?
      ii. What is the voltage across the capacitor?
      iii. What is the voltage across the resistor?
      iv. What is the current?

4. A 10 μF capacitor is charged to a voltage of 2 V. If it is connected to a 10 kΩ resistor, how long will it take to lose half of its charge on each plate?

5. The charged capacitor shown below has a positive plate on the left and negative plate on the right.



   In what direction does current flow through the resistor?

6. Suppose that a capacitor C_2 has twice the area as another capacitor C_1. Which capacitor can store more charge, if connected to identical batteries?

   (a) C_1
   (b) C_2
   (c) Neither; they can store the same amount of charge.

7. Suppose that a capacitor C_2 has twice the distance between the plates as another capacitor C_1. Which capacitor can store more charge, if connected to identical batteries?

   (a) C_1

   (b) C_2

   (c) Neither; they can store the same amount of charge.

8. Two 10 μF capacitors are connected in parallel. What is their effective capacitance?

9. Two 10 μF capacitors are connected in series. What is their effective capacitance?