CH21-1 – Waves

Important Ideas

- transverse wave, longitudinal wave
- sinusoidal wave
- wave function
- amplitude, frequency, period, wavelength, wavenumber
- Relationship of wavelength, frequency, and wave speed

$$v = \lambda f$$

- For a transverse wave on a string:

$$v = \sqrt{\frac{F_T}{\mu}}$$

- Wave function for a sinusoidal wave:

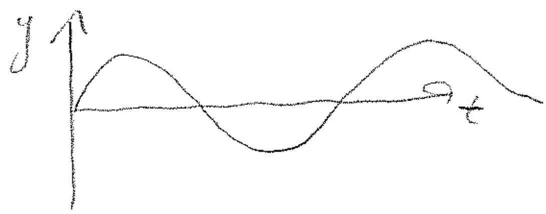
$$\begin{aligned} y(x,t) &= A\cos(\omega t \pm kx) \\ k &= 2\pi/\lambda \qquad \text{wavenumber} \\ \omega &= 2\pi f = \frac{2\pi}{T} \end{aligned}$$

Examples

1. A transverse wave on a string has a frequency of 220 Hz.
   (a) Describe the motion of a piece of the string.
   (b) Sketch a y vs. t graph for the piece of string.
   (c) How long does it take for a piece of the string to move from its lowest point to its highest point as the wave travels down the string?

(a) A piece of the string oscillates up and down in simple harmonic motion, like a mass on a spring

(b)



Assuming it's a sinusoidal transverse wave.

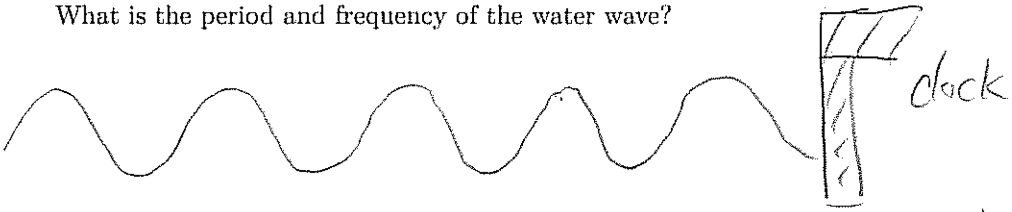
(c) 1 T is the time for 1 cycle.

$T = \frac{1}{f} = \frac{1}{220Hz} = 0.00455 s$

From the lowest point to the highest point is $\frac{1}{2}$ cycle so

$\Delta t = \frac{1}{2} T = 0.00227 s$

2. You toss a rock in a pond and observe the transverse waves the emanate from where the rock hit the water. You notice that the time required for 5 consecutive peaks to hit the post of the dock is 1.5 s. What is the period and frequency of the water wave?



Δt between peaks $= 1T$ $\qquad$ Δt for 4 cycles $= 1.5\,s$

5 peaks is 4 cycles $\qquad$ $T = \frac{1.5s}{4} = \boxed{0.375s}$ $\qquad$ $f = \frac{1}{T} = \boxed{2.67\,Hz}$

3. A wave on a guitar string oscillates with a frequency of 240 Hz. The distance between wave crests is 0.5 m. How fast does the wave travel down the string? If the wave is sinusoidal and travels to the left with an amplitude 0.5 mm, write an equation that describes the motion of a point on the string at location x and time t. What is the vertical position y for a point at $x = 0.2$ m at $t = 0$?

$f = 240\,Hz$

$\lambda = 0.5\,m$

$v = \lambda f = (0.5m)(240Hz) = \boxed{120\,\frac{m}{s}}$

$k = \frac{2\pi}{\lambda} = 12.6\,m^{-1}$

$y = A\cos(\omega t \pm kx)$ use + for left traveling wave

$\boxed{y = (5\times10^{-4}m)\cos(1508\,\frac{rad}{s}t + 12.6x)}$

$A = 0.5mm = 0.5\times10^{-3}m = 5\times10^{-4}m$

$\omega = 2\pi f = 2\pi(240Hz) = 1508\,\frac{rad}{s}$

at $t=0$, $x = 0.2m$

$y = 5\times10^{-4}\cos(12.6(0.2m)) = \boxed{-4.06\times10^{-4}m}$

Use rad mode for your calc.

4. Suppose that the string in the previous question has a linear density of 7 g/m. What is the tension in the string?

$\mu = 7\frac{g}{m} = 7\times10^{-3}\frac{kg}{m}$

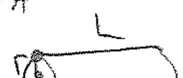
$v = \sqrt{\frac{F_T}{\mu}}$

$F_T = v^2\mu = (120\frac{m}{s})^2(7\times10^{-3}\frac{kg}{m}) = \boxed{101\,N}$

5. Two steel guitar strings have the same length. String A has a diameter of .5 mm and is under 410N of tension. String B has a diameter of 1.0 mm and the same tension. Find the ratio of the wave speeds, v_a/v_b, in these two strings.

$d_B = 2d_A$

$\rho = \frac{m}{V}$



$V = \pi r^2 L$

$m = \rho V = \rho\pi r^2 L$

so $\frac{m}{L} = \rho\pi r^2 = \mu$

$\mu \propto r^2$

so since $r_B = 2r_A$, then $\mu_B = 2^2\mu_A$

$\boxed{\mu_B = 4\mu_A}$

$v = \sqrt{\frac{F_T}{\mu}}$

$v \propto \frac{1}{\sqrt{\mu}}$ so $v_B = \frac{1}{\sqrt{4}}v_A$

6. For a guitar string of constant tension, the speed of the wave is constant. If you decrease the wavelength of a wave on the string by a factor of 2, by what factor does the frequency change?

(a) 2
(b) 1/2
(c) $\sqrt{2}$
(d) $1/\sqrt{2}$

$v = \lambda f$ = constant

$(\frac{1}{2}\lambda)(2f)$ f increases by factor of 2.

7. Suppose that you stretch a long spring between you and a friend. You shake one end to produce sinusoidal transverse waves. If you shake your hand twice as fast:

(a) What happens to the speed of the wave? stays the same
(b) What happens to the frequency of the wave? increases by 2
(c) What happens to the wavelength of the wave? changes by $\frac{1}{2}$

v depends on the medium, not the f

8. The wavelength of red light, near the edge of the visible region of the electromagnetic spectrum, is approximately 700 nm. What is the frequency of this light?

$v = \lambda f$

$(3\times10^{8}\,\frac{m}{s}) = (700\times10^{-9}\,m)f$

$f = 4.29\times10^{14}$ Hz

9. Suppose that a cell phone is tuned to send (and receive) radio waves of frequency 800 MHz. What is the wavelength of this radio wave?

$v = \lambda f$

$\lambda = \frac{v}{f} = \frac{3\times10^{8}\,\frac{m}{s}}{800\times10^{6}\,Hz} = 0.375\,m$

10. For sound in air of frequency 200 Hz, what is its wavelength? (Sound in air travels at about 343 m/s.)

$v = \lambda f$ $\quad f = 200$ Hz

$\lambda = \frac{v}{f} = \frac{343\,\frac{m}{s}}{200\,Hz} = 1.72\,m$