

Video Analysis of a Bicycle Wheel Gyroscope

Apparatus

Tracker software (free; download from <http://www.cabrillo.edu/~dbrown/tracker/>)
video: `gyroscope-300fps.mov` from <http://physics.highpoint.edu/~atitus/videos/>

Goal

In this experiment, you will use video analysis to measure the rotational period and precessional period of a bicycle wheel gyroscope. It is assumed that you know how to use the Tracker software.

Introduction

A gyroscope rotates counterclockwise about its center of mass (CM) when viewed looking down at the gyroscope wheel, toward the pivot. As a result, its angular momentum vector points outward, as shown below.

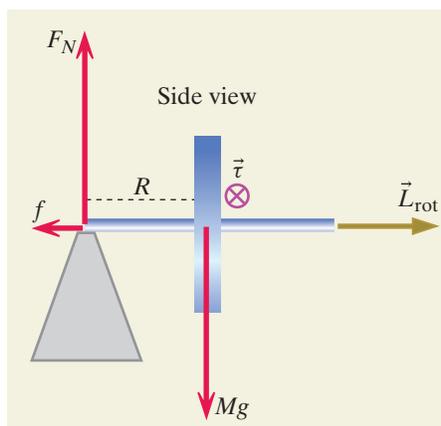


Figure 1: Side view of the gyroscope.

The torque about the pivot due to the gravitational force on the wheel is in the $-z$ direction. This means that $\Delta\vec{L}$ during a small time interval is also in the $-z$ direction. If you add $\Delta\vec{L}$ to the head of \vec{L} , then the new angular momentum will have a component that is into the page, causing the gyroscope to precess into the page. From a top view, this precession would be counterclockwise as shown below.

The angular precessional frequency Ω is

$$\Omega = \frac{RMg}{I\omega}$$

Procedure

Measuring the rotational angular speed

1. Download the video `gyroscope-300fps.mov`. This video was recorded at 300 fps.
2. Open *Tracker* and import the video.

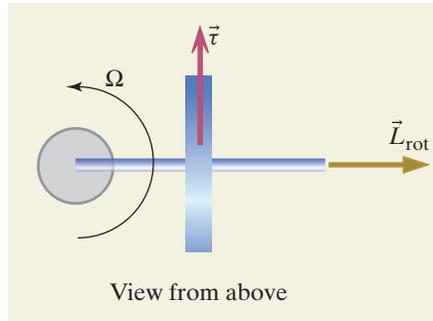


Figure 2: Top view of the gyroscope.

3. Open the Clip Settings window by clicking on the **Clip Settings** icon (shown in Figure 3) which is part of the video control toolbar that is below the video clip.



Figure 3: The Clip Settings icon.

4. In the **Clip Settings** pop-up window, enter a frame rate of 300 fps, as shown in Figure 4

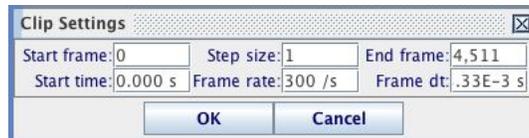


Figure 4: The Clip Settings icon.

5. A meterstick is shown in the first frame of video. Use this to calibrate distance in the video.
6. Since there are 300 frames of video recorded per second, the time interval between frames is quite small. As a result, we can skip frames between marking the ball and thus take fewer data points. Click on the **Step Size** button, as shown in Figure 5 and change it to **5**.

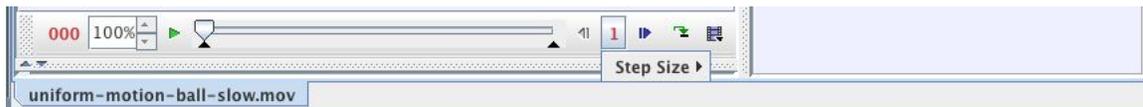


Figure 5: Change the step size in order to skip frames.

7. Step forward to the first frame that the wheel is rotating. Place the origin at the axle (at the center of the closest white disk at the hub of the wheel in the video).
8. Click the **Create** button in the toolbar and create a new point mass.

- Advance to the first frame that the wheel is rotating. Hold the shift key down and click once on the green sticker that is on the bicycle tire. The video will then advance 5 frames. Continue to mark the green sticker for a total of at least 3 revolutions. If you are only displaying the last few marks, then it will look something like Figure 6.



Figure 6: Marks for the green sticker on the rotating bicycle.

- Right-click on the x vs. t graph and select **Analyze**.
- Click the button **Fit Builder...**, click the button **Clone**, and select "Sinusoid." In the **Fit Builder** window, add a parameter d with value 0 and add a function $a * \sin(b * t + c) + d$, as shown in 7. The additive constant will shift the curve fit up and down by a constant.

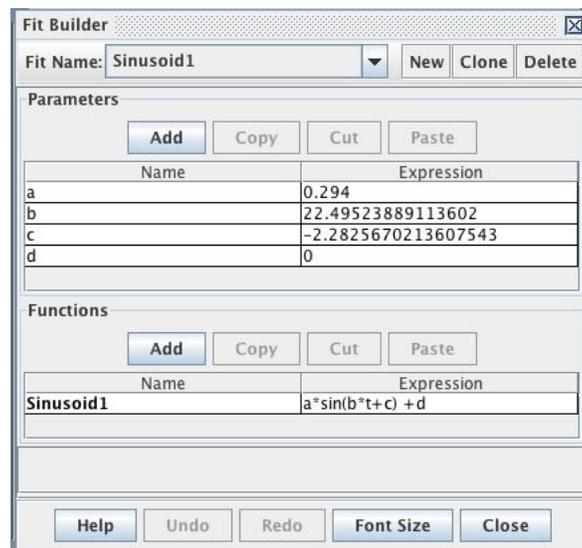


Figure 7: The Fit Builder is used to fit a curve to a user-defined function.

- Close the Fit Builder window.

13. You will need to adjust each parameter manually in order to fit a curve to x vs. t . It helps to start with some reasonable values. So make some approximate guesses for the following functions based on the curve:

a is the maximum value on the curve. What is approximately the maximum value of x ?

b is the angular velocity, $2\pi/T$. What is the approximate period of the oscillation? Use it to calculate an approximate value of b .

c is the phase. It shifts the curve right or left. You can adjust this manually without knowing the initial value.

d shifts the curve up or down. It should be small, less than 0.01.

If you begin with approximate values, you can click in the Parameter Values box and click the up and down arrows to make small adjustments. You can also change the step size as you hone in on the best value for the parameter.

An example curve-fit is shown in Figure 8.

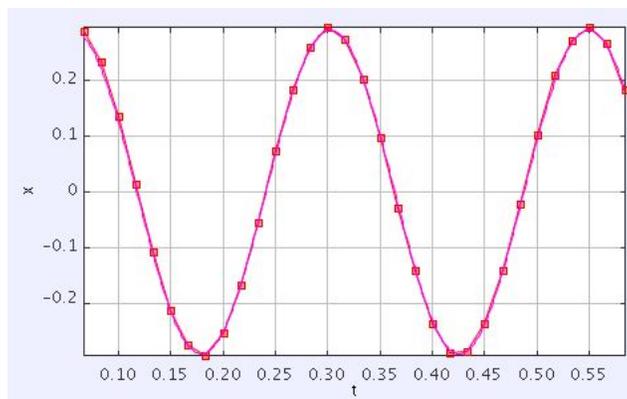


Figure 8: The best-fit curve for a point on the wheel.

What are the curve parameters and the fit equation for the best-fit curve?

From the curve fit, determine the radius of the wheel R_{wheel} and the angular velocity ω of the wheel.

How does your result compare to the actual radius of the wheel which is 30 cm?

Measuring the precessional angular speed

1. Start a new Tracker file and import the same video.
2. Calibrate the distance just like you did before.
3. Now, play the video all the way through and observe the precessional motion of the wheel. We want to measure the precessional period and frequency.

Through how many revolutions did it precess?

4. Change the video slider to when the wheel has precessed half a revolution (so that the string is in front of the plane of the wheel).
5. Set the origin of the coordinate system at the center of the white disks.
6. Click the button and add a point mass.
7. Mark the center of the white disk.
8. Play the video until the wheel has turned exactly 1/4 of a revolution. Mark the outer white disk (the one furthest to the left). This is the disk that we will track throughout its motion.
9. Play the video until the wheel has turned another 1/4 of a revolution. Now, the white disk is furthest to the right. Mark this disk.
10. Continue marking this disk for a total of 2 revolutions of the wheel.
11. Right-click the graph of x vs. t for mass B and select **Analyze**. Create a sinusoidal function of the form $x = a * \sin(b * t + c) + d$ and adjust the curve fit parameters manually until you get a decent curve fit. Then, select the checkbox for Autofit. It helps to turn off the "connect the dots" feature by clicking on the line style icon in the datatable. The resulting curve fit may look something like Figure 9.

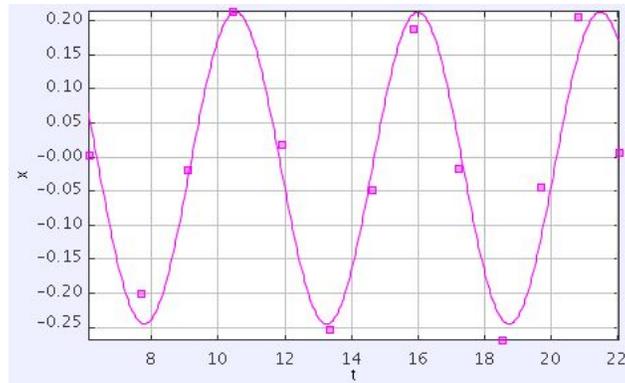


Figure 9: The best-fit curve for the precessional motion of the disk.

Approximate the period by taking the time of the last data point and subtracting the time for the first data point. Then, use $T = \Delta t / N$ where N is the number of revolutions during this time interval to calculate the period. Using this method, what is the period?

What is the equation and fit parameters for the best-fit curve?

What is the precessional angular velocity?

What is the period (as measured from the precessional angular velocity)?

Compare this result for the period to what you found by calculating $T = \Delta t/N$ where N is the number of revolutions during the time interval. It's probably best to report the period found from the curve fit, if you are reasonably satisfied that the curve fit indeed fits the data.

From the curve fit, determine the radius of precession for this particular white disk. Note that if the video were not so dark, we could measure this more precisely on the video itself using the “tape measure” tool. However, it can also be easily measured on the wheel itself, as shown below.

We really want to calculate the radius of precession from the point of support (the string) to the center of mass of the wheel. An end view of the axle and hub is shown in Figure 10.

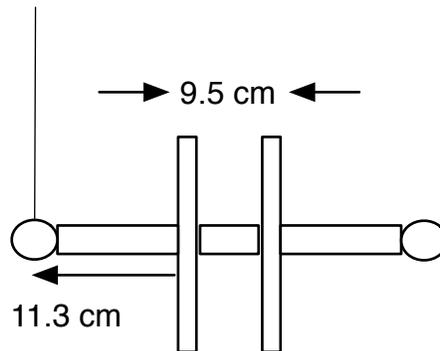


Figure 10: A side view of the axle and hub when the plane of the wheel first faces the camera.

Using the picture in Figure 10, calculate the radius of precession.

From your graph of x vs. t for the outer white disk, you determined the distance from the outer white disk to the string. Compare your result to what you measure from the picture shown in Figure 10. Explain the discrepancy in your results.